On the Use of Linear Programming Model Approach in Profit Optimization of a Product Mix Company

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Abstract

This study employed the utilization of a s*lack starting solution* approach of the Simplex Optimization method to build a Linear Programming (LP) model. Data were extracted from the record unit for an item blend fabricating industry, Fortunate Bakery, Ilorin, Nigeria. The information was collected on four noteworthy sorts of bread produced by the bakery which includes; Special Delight (SD), Ordinary Slice Bread (OSB), Ordinary Gala (OG) and Saloon (S) regarding their unit costs and selling prices, the raw materials utilized and the amount of each of the crude materials held in stock every day. In view of the data supplied, a linear programming problem was developed with the goal of maximizing the daily profit of the organization. The optimal daily profit that would be achievable to the organization in the item blend was resolved utilizing the methodology specified earlier. The results showed that the organization would achieve ideal daily profit level of ₦ 9,500 (or monthly profit level of ₦285,000) if she concentrates on the production of type alone is given to the unit offers of Saloon bread and disregard other lines of items produced by the company. By this, aggregate daily offers of about 380 (or monthly offers of about 11,400) units of Saloon bread would be sold by the organization. The data analyses were carried out USING *Tora* software package.

Keywords: Objective Function, Product Mix, Constraints, LP, Slack Variable, Tora.

Introduction

Businesses anywhere in the world including Nigeria are often confronted with inadequacies of some necessary materials which consequently lead to reduction in capacity utilization. As a result, optimal utilization of few available resources to achieve maximal benefits to the organization becomes paramount. Therefore, firm's Directors are continually confronted with how to make right choices among competing alternatives in order to meet their targets (objectives) which is either of maximizing the profit or minimizing the costs.

The development of businesses puts weight on administration in finding the optimal planning, organizing, leading and controlling levels of production in the various production industries of the economy. As a consequence of this pressure, administrative hypotheses of the firm are acquainted with investigating business situations and to take care of handy business issues like operational issues radiating from inside the business and ecological issues in which industry works. Both hypothetical and quantitative procedures are created to demonstrate and break down these choice issues technically known as decision problems. Among these quantitative procedures is the linear programming (LP) model, which utilizes numerical techniques as a part of searching for the optimal course of action in any decision situation under the restriction of limited resources and uncertainties.

The LP method proposed by Kantorovich (1960) is a mathematical technique used in operations research or management sciences to solve specific types of problems such as allocation, transportation and assignment problems that permit a choice between alternative courses of action. Linear Programming as described in Yahya (2004) can be characterized as a scientific system for deciding the best allocation of a company's restricted assets to accomplish ideal objective. As reported in Yahya *et al.* (2012), LP is one of the most viable techniques in operations research which is designed for linear objective functions and constraints. LP is a term that covers an entire scope of numerical methods that is meant for enhancing tasks execution regarding blend of assets (Lucey, 1996). Application of LP techniques has been demonstrated in literature such as; Albright and Wayne (2009), Byrd and Moore (1978), Lee and Plenert (1996), Chung *et al*. (2005), Savsar and Aldaihan (2011) and Yahya *et al* (2012) among others.

The objective of any linear programming problem is either to maximize the profit of an organization which is the case in this work or to minimize the cost of production with respect to the firm's limited resources. Dantzig (1963) provided an algorithm known as Simplex method for solving a linear programming problem. A – slack starting solution approach of the simplex algorithm requires the introduction of only the slack variables in a maximization problem. This allows determining the starting basic feasible solution in the simplex algorithm and the algorithm proceeds until the process terminates at optimal iteration level.

This work shows the utilization of linear programming methods in a production company in Nigeria. The issue attended to in this study was to decide the item blend (mix of products) to be focused on by the organization for production purpose at which the optimal profit level would be achieved.

Methodology

The data used for this work included four unique varieties of bread produced by Fortunate Bakery, Ilorin, Nigeria, as well as their cost prices and selling prices. These varieties included Special Delight (SD), Ordinary Slice Bread (OSB), Ordinary Gala (OG) and Saloon (S). Information on the raw materials include; flour measured in kilogram, sugar measured in gram, salt in gram, nectar measured in milliliter, flavor measured in unit, yeast in gram, spread in kilogram, water in liter, nutmeg in gram, additive in gram, conditioner in gram, groundnut oil in liter, machine time in minutes, work labour (in minutes) was also obtained.

Table 1: Unit Cost, Selling Prices and Unit profit for each of the Products

Products	Unit Cost of Production (₦)	Unit Selling Price (₦)	Unit Profit (₦)	% Unit Profit
Special Delight	120	200	80	40
Ordinary Slice Bread	90	150	60	40
Ordinary Gala	80	120	40	33.33
Saloon	35	60	25	41.67

The analysis was completed utilizing linear programming methods by developing a linear programming problem from the information obtained from the company. The linear programming issue created here is a numerical project in which both the objective function and constraints are linear in the unknown decision variables. Table 1 presents the unit cost of producing each of the four types of bread and their respective unit selling price.

Table 2: Quantity of Raw Materials used per Unit Product

Materials	Special Delight	Ordinary Slice Bread	Ordinary Gala	Saloon
Flour (kg)	0.4717	0.3876	0.2463	0.1316
Sugar (g)	61.3208	50.3876	32.0197	17.1053
Salt (g)	14.1509	11.6279	7.3892	3.9474
Honey (ml)	4.7170	3.8760	2.4631	1.3158
Flavour (unit)	0.2830	0.2326	0.1478	0.0789
Yeast (g)	4.7170	3.8760	2.4631	1.3158
Butter (kg)	0.0094	0.0078	0.0049	0.0026
Water (liter)	0.3019	0.2481	0.1576	0.0842
Nutmeg (g)	0.1887	0.1550	0.0985	0.0526
Preservative (g)	1.3208	1.0853	0.6897	0.3684
Softener (g)	0.2358	0.1938	0.1232	0.0658
Groundnut oil (liter)	0.0094	0.0078	0.0049	0.0026
Machine (min)	0.3302	0.2713	0.1724	0.0921
Labour (min)	7.9245	6.5116	4.1379	2.2105

Table 2 provides information on quantity of raw materials used for each of the product lines and Table 3 contains the stock capacity at production line for each of the raw materials used in the production of the four products.

Table 3: Stock Capacity of the Raw Materials

Materials	Quantity Available in Stock
Flour	50kg
Sugar	6500g
Salt	1500g
Honey	500ml
Flavor	30unit
Yeast	500g
Butter	1kg
Water	32lt
Nutmeg	20g
Preservative	140g
Softener	25g
Groundnut oil	1 liter
Machine	35min
Labour	14hrs (840min)

The general form of a linear programming problem as used in this work is stated below:

$$Maximize\ z = c_1x_1 + c_2x_2 + c_3x_3 + \ \dots + c_nx_n \quad (1)$$

$$\left.\begin{aligned} Subject\ to{:}\ & a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n \le b_1 \\ & a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n \le b_2 \\ & \quad .. \qquad .. \qquad .. \qquad .. \\ & \quad .. \qquad .. \qquad .. \qquad .. \\ & a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n \le b_m \end{aligned}\right\} \quad (2)$$

$$\text{for } x_1,\ x_2, x_3, \dots x_n \ge 0 \quad (3)$$

Every linear programming problem is characterized by equations (1), (2) and (3) above referred to as the objective function, constraints and non–negativity restrictions respectively. Where $x_1,\ x_2, \dots\ x_n$ are called decision variables of the problem, $c_1,\ c_2, \dots\ c_n$ are the known contributions of the decision variables respectively, $b_i\ (for\ i = 1, 2, \dots, m)$ are the amount of the raw materials available for each product mix and $a_{ij}\ (for\ i = 1, 2, \dots, m\ \ and\ \ j = 1, 2, \dots, n)$ are called the technological coefficients.
For the purpose of using the all-slack starting solution approach, a standard linear programming model would be determined by adding slack variables into the model. The standard linear programming model is then given as:

$$Maximize\ Z - c_1x_1 - c_2x_2 - c_3x_3 - \ \dots - c_nx_n = 0 \quad (4)$$

$$\left.\begin{aligned} Subject\ to{:}\ & a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n + s_1 = b_1 \\ & a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n + s_2 = b_2 \\ & \quad .. \qquad .. \qquad .. \qquad .. \quad . \\ & \quad .. \qquad .. \qquad .. \qquad .. \\ & a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n + s_m = b_m \end{aligned}\right\} \quad (5)$$

$$\text{for } x_1, x_2, x_3, \dots x_n,\ s_1, s_2, \dots s_m \ge 0 \quad (6)$$

Where $s_i\ (i = 1, 2, \dots, m)$ are called the slack variables.

The whole analysis was performed by utilizing the TORA software package developed by Hamdy (2007).

Data Analysis

The data obtained from Fortunate Bakery, Ilorin, Nigeria was analyzed to determine the best product mix of the company that will maximize the daily (monthly) profit for the organization. We represent each of the company's product lines, Special Delight, Ordinary Slice Bread, Ordinary Gala and Saloon, by $x_1,\ x_2,\ x_3\ and\ x_4$ respectively. Hence, the linear programming model was formulated as:

$Maximize\ Z = 80x_1 + 60x_2 + 40x_3 + 25x_4$ (7)

$Subject\ to$: (raw materials constraints)

Flour: $0.4717x_1 + 0.3876x_2 + 0.2463x_3 + 0.1316x_4 \leq 50$

Sugar: $61.3208x_1 + 50.3876x_2 + 32.0197x_3 + 17.1053x_4 \leq 6500$

Salt: $14.1509x_1 + 11.6279x_2 + 7.3892x_3 + 3.9474x_4 \leq 1500$

Honey: $4.7170x_1 + 3.8760x_2 + 2.4631x_3 + 1.3158x_4 \leq 500$

Flavour: $0.2830x_1 + 0.2326x_2 + 0.1478x_3 + 0.0789x_4 \leq 30$

Yeast: $4.7170x_1 + 3.8760x_2 + 2.4631x_3 + 1.3158x_4 \leq 500$

Butter: $0.0094x_1 + 0.0078x_2 + 0.0049x_3 + 0.0026x_4 \leq 1$

Water: $0.3019x_1 + 0.2481x_2 + 0.1576x_3 + 0.0842x_4 \leq 32$ (8)

Nutmeg: $0.1887x_1 + 0.1550x_2 + 0.0985x_3 + 0.0526x_4 \leq 20$

Preservative: $1.3208x_1 + 1.0853x_2 + 0.6897x_3 + 0.3684x_4 \leq 140$

Softener: $0.2358x_1 + 0.1938x_2 + 0.1232x_3 + 0.0658x_4 \leq 25$

Groundnut oil: $0.0094x_1 + 0.0078x_2 + 0.0049x_3 + 0.0026x_4 \leq 1$

Machine: $0.3302x_1 + 0.2713x_2 + 0.1724x_3 + 0.0921x_4 \leq 35$

Labour: $7.9245x_1 + 6.5116x_2 + 4.1379\,x_3 + 2.2105x_4 \leq 840$

for $x_1,\ x_2,\ x_3,\ x_4 \geq 0$ (9)

In order to write the above linear programming model in standard form, fourteen (14) slack variables, x_i $(for\ i = 5\ to\ 18)$ were introduced into the model as stated earlier and the introduction of these slack variables changes the inequality signs in the constraints to an equation. Hence, the set of inequalities in (7), (8) and (9) respectively become (10), (11) and (12).

$Maximize\ Z - 80x_1 - 60x_2 - 40x_3 - 25x_4 = 0$ (10)

$subject\ to$: (Raw materials constraints)

Flour: $0.4717x_1 + 0.3876x_2 + 0.2463x_3 + 0.1316x_4 + x_5 = 50$

Sugar: $61.3208x_1 + 50.3876x_2 + 32.0197x_3 + 17.1053x_4 + x_6 = 6500$

Salt: $14.1509x_1 + 11.6279x_2 + 7.3892x_3 + 3.9474x_4 + x_7 = 1500$

Honey: $4.7170x_1 + 3.8760x_2 + 2.4631x_3 + 1.3158x_4 + x_8 = 500$

Flavour: $0.2830x_1 + 0.2326x_2 + 0.1478x_3 + 0.0789x_4 + x_9 = 30$

Yeast: $4.7170x_1 + 3.8760x_2 + 2.4631x_3 + 1.3158x_4 + x_{10} = 500$

Butter: $0.0094x_1 + 0.0078x_2 + 0.0049x_3 + 0.0026x_4 + x_{11} = 1$

Water: $0.3019x_1 + 0.2481x_2 + 0.1576x_3 + 0.0842x_4 + x_{12} = 32$ (11)

Nutmeg: $0.1887x_1 + 0.1550x_2 + 0.0985x_3 + 0.0526x_4 + x_{13} = 20$

Preservative: $1.3208x_1 + 1.0853x_2 + 0.6897x_3 + 0.3684x_4 + x_{14} = 140$

Softener: $0.2358x_1 + 0.1938x_2 + 0.1232x_3 + 0.0658x_4 + x_{15} = 25$

Groundnut Oil: $0.0094x_1 + 0.0078x_2 + 0.0049x_3 + 0.0026x_4 + x_{16} = 1$

Machine: $0.3302x_1 + 0.2713x_2 + 0.1724x_3 + 0.0921x_4 + x_{17} = 35$

Labour: $7.9245x_1 + 6.5116x_2 + 4.1379\,x_3 + 2.2105x_4 + x_{18} = 840$

for $x_1,\ x_2,\ x_3,\ \ldots,\ x_{18} \geq 0$ (12)

Generally, the slack variables introduced in any linear programming model represent the level of unused resources or raw materials if such occurs at the optimal solution of the simplex algorithm. This new representation generates the initial tableau in Table 4 as used for our LP implementation.

Table 4: Initial Tableau of the Linear Programming Model

Basis	**X_1**	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**X_7**	**X_8**	**X_9**	**X_{10}**	**X_{11}**	**X_{12}**	**X_{13}**	**X_{14}**	**X_{15}**	**X_{16}**	**X_{17}**	**X_{18}**	**Sol**
Z	−80	−60	−40	−25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_5	0.4717	0.3876	0.2463	0.1316	1	0	0	0	0	0	0	0	0	0	0	0	0	0	50
X_6	61.3208	50.3876	32.0197	17.1053	0	1	0	0	0	0	0	0	0	0	0	0	0	0	6500
X_7	14.1509	11.6279	7.3892	3.9474	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1500
X_8	4.7170	3.8760	2.4631	1.3158	0	0	0	1	0	0	0	0	0	0	0	0	0	0	500
X_9	0.2830	0.2326	0.1478	0.0789	0	0	0	0	1	0	0	0	0	0	0	0	0	0	30
X_{10}	4.7170	3.8760	2.4631	1.3158	0	0	0	0	0	1	0	0	0	0	0	0	0	0	500
X_{11}	0.0094	0.0078	0.0049	0.0789	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
X_{12}	0.3019	0.2481	0.1576	0.0842	0	0	0	0	0	0	0	1	0	0	0	0	0	0	32
X_{13}	0.1887	0.1550	0.0985	0.0526	0	0	0	0	0	0	0	0	1	0	0	0	0	0	20
X_{14}	1.3208	1.0853	0.6897	0.3684	0	0	0	0	0	0	0	0	0	1	0	0	0	0	140
X_{15}	0.2358	0.1938	0.1232	0.0658	0	0	0	0	0	0	0	0	0	0	1	0	0	0	25
X_{16}	0.0094	0.0078	0.0049	0.0026	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
X_{17}	0.3302	0.2713	0.1724	0.0921	0	0	0	0	0	0	0	0	0	0	0	0	1	0	35
X_{18}	7.9245	6.5116	4.1379	2.2105	0	0	0	0	0	0	0	0	0	0	0	0	0	1	840

Table 5: Values of the Unused Raw Materials at Optimal Iteration Level

Slack Variable	**Corresponding Raw Material**	**Unused Amount**
X_5	Flour (kg)	0
X_6	Sugar (g)	1.03
X_7	Salt (g)	0.23
X_8	Honey (ml)	0.08
X_9	Flavor (unit)	0.02
X_{10}	Yeast (g)	0.08
X_{11}	Butter (kg)	0.01
X_{12}	Water (liter)	0.01
X_{13}	Nutmeg (g)	0.02
X_{14}	Preservative (g)	0.03
X_{15}	Softener (g)	0
X_{16}	Groundnut oil (liter)	0.01
X_{17}	Machine (min)	0.01
X_{18}	Labour (min)	0.14

Table 6: Final Tableau of the Linear Programming Model

Basis	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	Sol.
Z	9.61	13.63	6.79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9498.48
X_{13}	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0.02
X_6	0.01	0.01	0.01	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1.03
X_7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0.23
X_8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0.08
X_9	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0.02
X_{10}	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0.08
X_{11}	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0.01
X_{12}	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0.01
X_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0.03
X_4	**3.58**	**2.95**	**1.87**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**379.94**
X_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0.00
X_{16}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0.01
X_{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0.01
X_{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.14

Findings and Discussion

From the analysis carried out on the linear programming model stated above using the all-slack starting solution of the Simplex method, the optimal solution for the LP was obtained at the fifth iteration. The daily estimated value of the objective function was obtained to be 9498.4802 (approximately 9,500) and the daily contributions of each of the four decision variables x_1, x_2, x_3, x_4 into the objective function are 0, 0, 0, and 379.94 (approximately 380). The values of the slack variables corresponding to the unused raw materials are provided in Table 5. The final optimization results as reported by our algorithm are presented in the final tableau of Table 6.

Conclusions

This work has clearly demonstrated the appropriate use of the all-slack starting solution approach of the simplex algorithm in linear programming model. This is obvious from the analysis and results obtained in the profit maximization type of the linear programming model fitted to the data collected on four different varieties of bread produced by Fortunate Bakery, Ilorin, Kwara State, Nigeria.

Based on the analysis carried out and the results obtained from the linear programming model, it is recommended that the Fortunate Bakery be focus more on the production of saloon bread based on the available daily raw materials. By this decision, a total number of 380 units of saloon bread would be produced and sold by the Bakery daily. This would allow the organization to realize an optimal daily profit of ₦9,500 based on the available raw materials. A very quick check of this daily realization is to divide the value of the objective function at optimal level (₦9,500) by the value of the decision variable (saloon bread) also at the same optimal level (380). This gives a unit profit of ₦25 (41.67%) per saloon bread and consequently, agrees perfectly with the expected daily profit of the company on a unit sale of the saloon bread as reported in Table 1. Results from Table 5 show that virtually all the raw materials are efficiently utilized in the daily production of the four varieties of bread produced by Fortunate Bakery. This suggests the importance of the raw materials collected in the production of the products.

Recommendations

Finally, since it is almost difficult for any researcher to impose an idea on an operating management of a system that one does not belong to, therefore, the results of the LP model in this work are only based on daily availability of raw materials for the production of the four types of bread produced by Fortunate Bakery which could serve as a guide to the management of the company in the production strategies of their products.

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