Magnetic Effects on Laminar Free Convective Heat and Mass Transfer over a Vertical Plate with Arrhenius Kinetics

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Abstract
The present paper investigated heat and mass transfer characteristics of free convection, steady two-dimensional laminar flow of an incompressible reacting fluid over a vertical plate under the influence of magnetic field, buoyancy and thermal conductivity. The governing partial differential equations of the problem, using similarity transformations, were reduced to a couple nonlinear ordinary differential equations and solved numerically by Runge-Kutta fourth order method with shooting technique. The solution was found to be dependent on several governing parameters, including the magnetic field parameter, Grashof number, Prandtl number, activation energy parameter and Frank-Kamenetskii parameter. The numerical results concerned with velocity and temperature profiles effects of various parameters on the flow field were investigated and presented in table and displayed graphically.

Keywords: Reacting flow; Convection; Thermal conductivity; Frank-Kamenetskii parameter; Activation energy.

Introduction

The study of flow problems of free convection under the influence of magnetic field has attracted many researchers in view of its application in geophysics, astrophysics, geological formations, and thermal recovery of oil, and in assessment of aquifers, geothermal reservoirs and underground nuclear waste storage site, etc. Also, the growing needs in industries and engineering, requires the study of heat and mass transfer in the presence of different conditions and parameters with reaction flow effects. It has many applications in nuclear reactor and combustion, solar collectors, drying, dehydration operations in chemical and food processing plants, polymer production, etc.

Omowaye and Ayeni (2009) studied unsteady MHD free convection flow and heat transfer along an infinite vertical porous plate under Arrhenius kinetics. Their studied showed that velocity of the fluid decreases with the increase in Prandtl number and Hartmann number but increases with increase in Grashof number. While, temperature decreases with increase in Prandtl number. Basant (1998), considered the effects of applied magnetic field on transient free-convective flow in a vertical channel. His result showed that as magnetic parameter increases, velocity decreases, while it increases with increase in time (t). Steady Arrhenius laminar free convective MHD flow and heat transfer past a vertical stretching sheet with viscous dissipation was studied by Omowaye and Koriko (2014) and their results indicated that velocity and temperature profile increases with increase in local Grashof number and Eckert number. Mansuor *et al.* (2008) considered a steady two-dimensional nonlinear MHD boundary layer flow of an incompressible, viscous and electrically conducting fluid in the presence of a uniform magnetic field with heat, mass transfer and chemical reaction in a porous medium. The fluid properties were assumed to be constant. The results showed that the flow field was influenced appreciably by the presence of chemical reaction, viscous dissipation and suction or injection flow.

Mamun *et al.* (2007) investigated combined effect of conduction and viscous dissipation on magnetohydrodynamic free convection flow along a vertical flat plate. Their investigation shows that velocity increases with increase in magnetic parameter, Prandtl number and conjugate conduction parameter while, temperature increases with increase in magnetic and dissipation parameter but decreases with increase in Prandtl number and conjugate conduction parameter. Amin (2003) investigated the effects of viscous dissipation on buoyancy-induced flow over a horizontal or a vertical flat plate embedded in a non-Newtonian fluid saturated porous medium under the action of transverse magnetic field. He used the Ostwald-de Waele power-law model to characterize the non-Newtonian fluid behaviour. Heat and mass transfer characteristics and flow behaviour on MHD flow near the lower stagnation point of a porous isothermal horizontal circular cylinder was studied by Ziya and Manoj (2011). Their result showed that velocity increases with viscosity parameter, while temperature decreases with the same parameter. Also, both velocity and temperature decrease with increase in radiation and increases with increase in thermal conductivity. Thermal criticality for a reactive gravity driven thin film flow of a third-grade fluid with adiabatic free surface down an inclined plane was studied by Makinde (2009). His investigation revealed that an increase in the material parameter enhances the thermal stability of the liquid and his series summation procedure can be used as an effective tool to investigate several other parameter-dependent nonlinear boundary-value problems in science and engineering. Analytical solutions for the problem of heat and mass transfer by steady flow of an electrically conducting and heat generating/absorbing fluid on a uniformly moving vertical permeable surface in the presence of a magnetic field of first order chemical reaction was studied by Chamkha (2003). His result showed that fluid velocity decreased as Prandtl number, the Schmidt number and the strength of the magnetic field was increased but increased as thermal and concentration buoyancy effects were increased. Similarly, heat and mass transfer of MHD free convection steady flow of an incompressible electrically conducting fluid over an inclined plate under the influence of an applied uniform magnetic field and the effects of Hall current was studied by Alam *et al.* (2013). Their results showed that velocity increases with increase in modified magnetic field but decreases with inclination of the plate, buoyancy ratio and magnetic parameter. As no effect of various parameters on secondary velocity, temperature and concentration was felt over the inclined plate. Recently, Mohyud-Din *et al.* (2010) investigated Modified Variational Iteration Method (MVIM) for free-convective boundary-layer equation using Padé approximation.

However, effects of magnetic field and the reacting fluid under Arrhenius kinetics with laminar free convection flow over a vertical plate has not been adequately dealt with, hence the present work. We further assume that the flow is subject to magnetic field and there is heat generation in the reacting flow.

Problem Formulation

A steady two-dimensional laminar free convection flow of a viscous, incompressible fluid over a vertical plate with reacting flow and buoyancy effects is considered. The x-axis is taken along the vertical plate in the upward direction and the y-axis is normal to the plate. The magnetic field is assuming to be applied. The flow field is governed by the following equations (Mohyud-Din *et al.*, 2010) given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma\beta_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{AQexp^{-\frac{E}{RT}}}{\rho C_p} \tag{3}$$

where u, v are the velocity components in x, y directions respectively. ρ density of the fluid, υ the kinematic viscosity, g the acceleration due to gravity, σ electrical conductivity, T the dimensional temperature, β and β_0 are the coefficient of volumetric expansion and magnetic field intensity.

The boundary conditions for the velocity and temperature fields are:

$$u=0 \quad v=0 \quad T=T_w \quad at \quad y=0 \tag{4}$$

$$u\to 0 \qquad T\to T_\infty \qquad as \qquad y\to\infty \tag{5}$$

where T_w is the wall dimensional temperature and T_∞ is free stream dimensional temperature. Introducing the stream function $\psi(x, y)$ as defined by

$$u=\frac{\partial\psi}{\partial y}\text{and} \quad v=-\frac{\partial\psi}{\partial x} \tag{6}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following dimensionless quantities are introduced

$$\eta = y\sqrt{\frac{U_0}{2\upsilon x}}\ ;\ f'(\eta) = \frac{u}{U_0}\ ;\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty};\ \psi = \sqrt{2x\upsilon U_0}\, f(\eta) \tag{7}$$

In view of equations (6) and (7), equations (2) – (3) reduces to the following couple nonlinear ordinary differential dimensionless form of equation;

$$f''' + ff'' + Gr\theta - Mf' = 0 \tag{8}$$

$$\theta'' + Prf\theta' + \ Pr\delta exp^{\left(\frac{\theta}{1+\varepsilon\theta}\right)} = 0 \tag{9}$$

where prime is the differentiation with respect to η, f is the dimensionless velocity, θ is the dimensionless temperature, $Gr = \frac{2xg\beta(T_w - T_\infty)}{U_0^2}$ is the local Grashof number, $Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number, $\varepsilon = \frac{RT_\infty}{E}$ is the activation energy parameter, $\delta = \frac{2xAQexp^{-\frac{E}{RT_\infty}}}{\rho C_p U_0(T_w - T_\infty)}$ is the Frank-Kamenetskii parameter and $M = \frac{2x\sigma\beta_0^2}{\rho U_0}$ is local Magnetic parameter of the flow.

The corresponding boundary conditions are

$$\left.\begin{array}{lll} f(0) = 0, & f'(0) = 0, & \theta(0) = 1 \\ f'(\infty) = 0, & \theta(\infty) = 0 & \end{array}\right\} \tag{10}$$

Hence, equations (8) and (9) subject to (10) are the local similarity equations governing the flow. Examining now, the skin-friction and Nusselt number which are important physical parameters for this type of boundary layer flow. The skin-friction over the plate, which is the dimensionless form, is given by

$$C_f = \frac{\tau_w}{\rho\upsilon} = \left(\frac{\eta}{y\rho\upsilon}\right) f''(0) \tag{11}$$

The rate of heat transfer coefficient, which is the dimensionless form in terms of the Nusselt number, is given by

$$Nu = \left(\frac{\eta}{y}\right)\frac{q_w}{k(T_w - T_\infty)} = -\theta'(0) \tag{12}$$

where $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and $q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}$ is the shear stress and rate of heat transfer over the plate.

Numerical Computation

The set of nonlinear coupled ordinary differential equations (8) and (9) with boundary conditions (10) are solved numerically using the Runge-Kutta fourth order scheme along with shooting method. The higher order nonlinear differential equations (8) and (9) are reduced to a system of first order differential equations as follows:

Let,

$$f = y_1,\ f' = y_2,\ f'' = y_3, \theta = y_4, \theta' = y_5 \tag{13}$$

Then from equation (13), the system of linear ordinary differential equations is obtained. Thus,

$$
\begin{aligned}
y_1' &= y_2 \\
y_2' &= y_3 \\
y_3' &= -y_1 y_3 - Gr y_4 + M y_2 \\
y_4' &= y_5 \\
y_5' &= -Pr y_1 y_5 - Pr\delta exp^{\left(\frac{y_4}{1+\varepsilon y_4}\right)}
\end{aligned}
\qquad (14)
$$

The behaviour of the physical parameters Gr, Pr, M, ε and δ are calculated numerically to determine the skin-friction coefficient $f''(0)$ and the Nusselt number $\theta'(0)$ as presented in table 1 and the results are presented in figures 1 – 10.

Result and Discussions

The problem of steady free convective flow of a viscous, incompressible flow with magnetic effect in the reacting flow over a vertical plate was solved by Runge-Kutta fourth order method with shooting technique. The expressions for the velocity and temperature were obtained. To illustrate the behaviour of these physical quantities on the velocity and temperature fields, numeric values were computed with respect to the variations in the governing parameters viz, the local Grashof number Gr, Prandtl number Pr, activation energy ε, local magnetic parameter M and Frank-Kamenetskii parameter δ and are analyzed with the help of graphs.

The effect of magnetic field parameter M on the velocity of the flow f' and temperature θ when $M = 1, 2, 8$, $Gr = 2$, $Pr = 0.72$, $\varepsilon = 0.1$ and $\delta = 0.01$ is illustrated in figures 1 – 2. From figure 1, it is observed that velocity decreases as the magnetic parameter increases whilst in figure 2, temperature θ increases as M increases in the vicinity of the plate. The effect of Grashof number Gr for heat transfer on the velocity of the flow field when $Gr = 2, 5, 10$, $M = 2$, $Pr = 0.72$, $\varepsilon = 0.1$ and $\delta = 0.01$, is presented in figure 3. It is shown that velocity increases along y with increasing values of Gr, which is due to enhancement in buoyancy force. In figure 4, the effect of Gr for heat transfer on the temperature shows that θ decreases with increasing values of Grashof number.

The effect of Prandtl is important in velocity and temperature profile. Figures 5 and 6 depict the effect of Prandtl number Pr on velocity and temperature distribution respectively. It is observed that velocity and temperature decrease with increase in Pr when $Pr = 0.72, 2, 4$, $M = 2$, $Gr = 2$, $\varepsilon = 0.1$ and $\delta = 0.01$. Figure 7 shows the velocity distribution for different values of activation energy ε. It is observed that velocity increases with increasing activation energy but the rate of heat transfer on temperature decreases with increase in ε as shown in figure 8. The effect of velocity and temperature profile for different values of Frank-Kamenetskii parameter is shown in figures 9 and 10. It is observed that velocity and temperature profiles increase with increasing Frank-Kamenetskii parameter δ. The effects of skin-friction (C_f) and Nusselt number (Nu) are numerically shown in table 1. From table 1, the numerical values of $f''(0)$ and $\theta'(0)$ for different values of dimensionless parameters Pr, Gr, M, ε and δ as presented above indicates that the dimensionless wall velocity gradient $f''(0)$ increases as Gr and δ increases while it decreases with increase in Pr, M and ε. Moreover, the value of $\theta'(0)$ decreases as Pr, ε and Gr increases while increase in M and δ increases the Nusselt number also.

Table 1: Numerical values of skin-friction (C_f) and Nusselt number (Nu)

Pr	Gr	M	ε	δ	C_f	Nu
0.72	2	2	0.1	0.01	1.1020	- 0.3298
4	2	2	0.1	0.01	0.9251	-

0.72	10	2	0.1	0.01	4.4598	0.5643
0.72	2	8	0.1	0.01	0.6589	-0.6036
0.72	2	2	9	0.01	1.0978	-0.1926
0.72	2	2	0.1	0.05	1.1927	-0.3388
						-0.1983

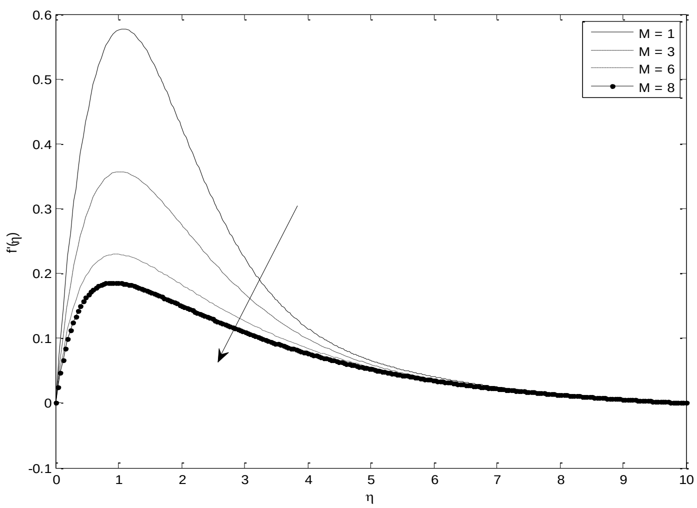

Figure 1: Velocity profile for different values of M

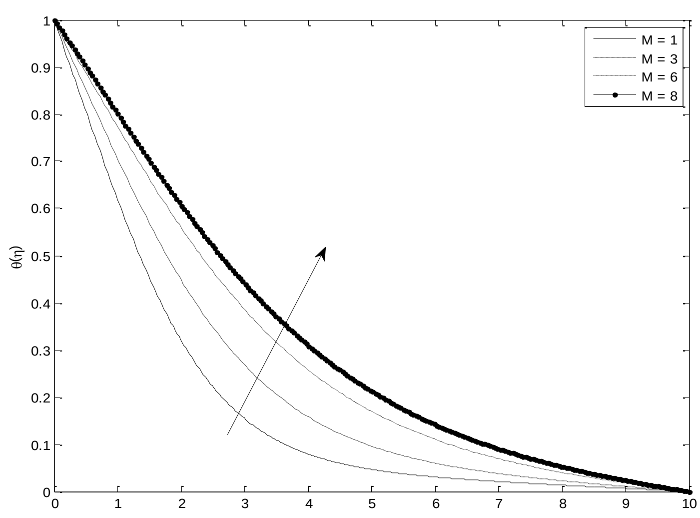

Figure 2: θ against various values of M

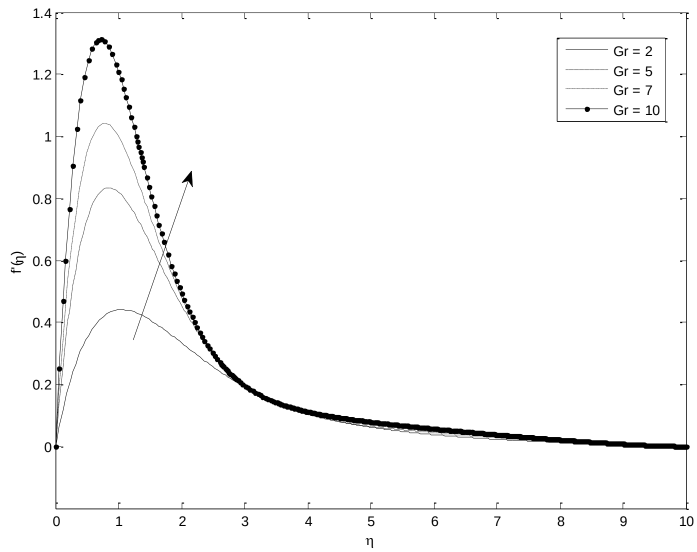

Figure 3:f' against various values of Gr

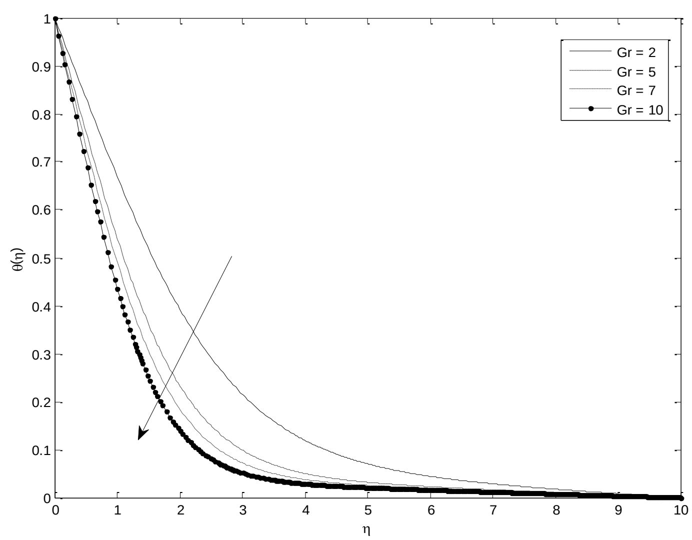

Figure 4: Temperature against different values of Gr

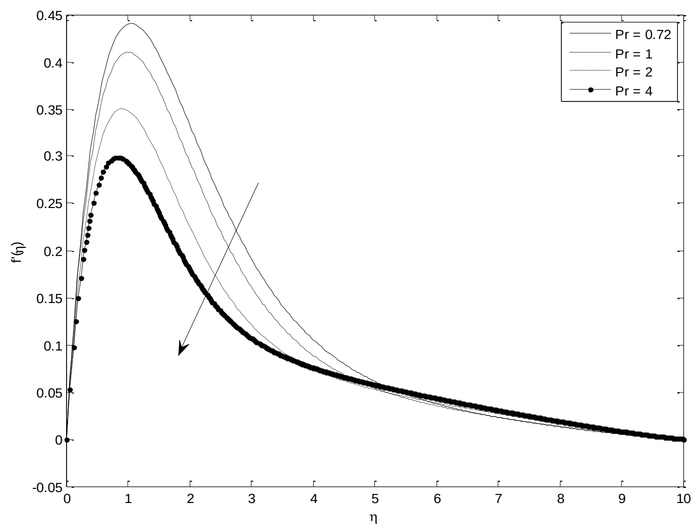

Figure 5: Velocity profile for different values of Pr

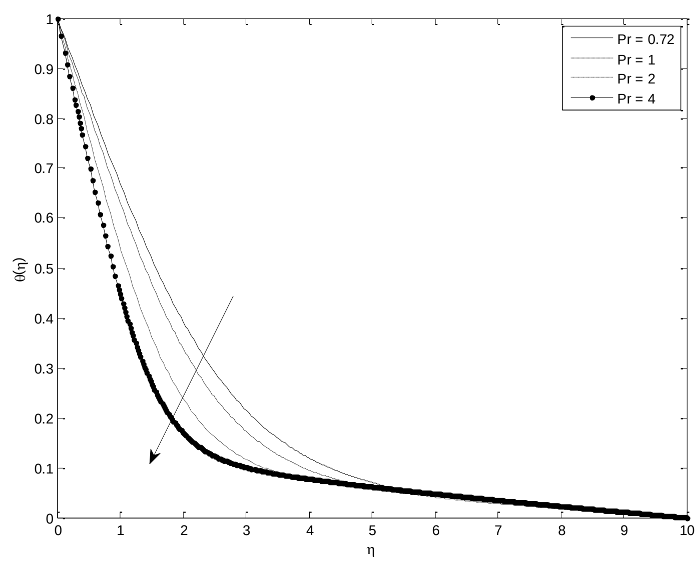

Figure 6: Temperature profile for various values of Pr

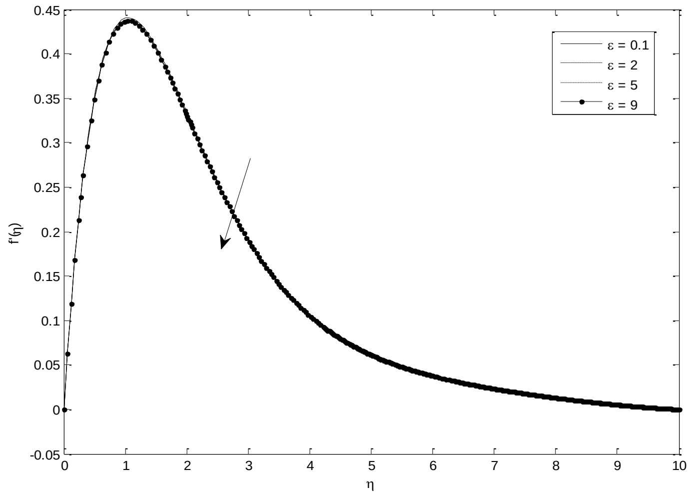

Figure 7: Velocity profile against different values of ε

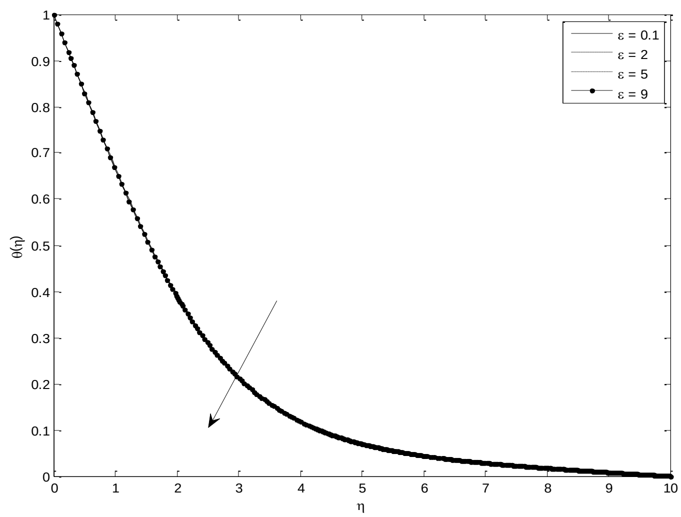

Figure 8: Temperature profile for different values of ε

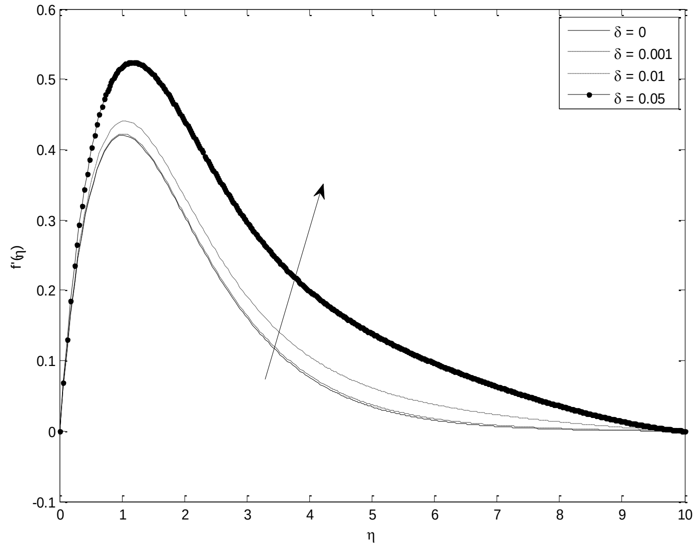

Figure 9: Velocity profile for various values of δ

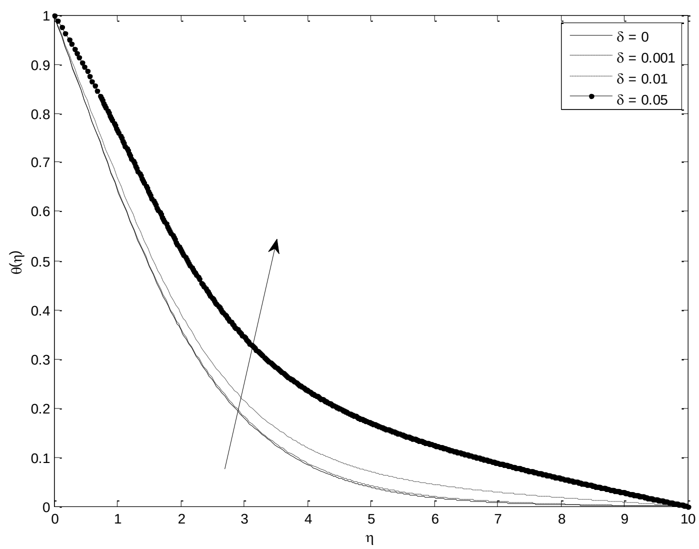

Figure 10: Temperature profile for various values of δ

Conclusions and Recommendations

In this paper, the magnetic effects of heat transfer on free convective flow over a vertical plate in the reacting flow have been studied and numerical results are presented to illustrate the details of the flow condition and fluid properties. From the investigation, the following conclusions are drawn; that, the Skin-friction increases with increase in local Grashof number and Frank-Kamenetskii parameter and decreases with increase in Prandtl number, magnetic parameter and activation energy. While, increase in Prandtl number, activation energy and Grashof number, leads to a decrease in the value of Nusselt number but increases with increase in magnetic and Frank-Kamenetskii parameter.

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