An Investigative Study of Magnetohydrodynamic Flow of a Third Grade Fluid through Porous Channel in the Presence of InducedMagnetic Field and Viscous Dissipation

O. W. Lawal [1] and L.M. Erinle-Ibrahim[2]
[1]Department of Mathematics, University of the Gambia
[2]Department of Mathematics, Tai Solarin University of Education Ijagun, Ogun State
waheedlawal207@yahoo.com

Abstract
This work investigates the MHD flow of a third-grade fluid in a porous channel under theinfluence of aninduced magnetic field and viscous dissipation. Perturbation technique is applied to analytically solvedthe set of coupled nonlinear ordinary differential equations that govern the flow and solutions expressionfor velocity field, temperature field and induced magnetic field are accomplished. The effects of nondimensional parameters on velocity field, induced magnetic field, heat transfer and skin friction aredisplayed inform of graphs and table. From the results, it is realised that the effects of suction parameter are to increase the velocity and temperature field while increases in magnetic parameter and magneticPrandtl number reduces the velocity field and increases the induced magnetic field.

Keywords: Induced Magnetic Field; Third Grade Fluid; Non-Newtonian Fluid; Skin Friction.

Introduction

The study of MHD flow of third grade fluid with the occurrence of an apparent external magnetic field has gained appreciable awareness in modern research because of its relevance in science, engineering and technology. Some of these applications can be found in materials manufactured by extraction process especially in polymer processing, micro fluids, geological flows within the earth's mantle, the flow of synovial fluid in human joints as well as in the drilling of oil and gas well. Hayat, Kara and Momoniat [1] studied the flow of a third fluid on a porous wall using Lie group method. They reduced the third order differential equation governing the flow to a second order differential equation, which is then solved using perturbation method.
Siddique, Zeb, Ghori, and Behaibt [2] studied hydrodynamics third grade fluid between to parallel plates with heat transfer. They considered and treated three different problems, the poiseuille flow, the couette flow and the poiseuille-couette flow. Aiyesimi, Okedayo and Lawal [3] extend this work (i.e. [2]) in two ways (i) by considered that the fluid is flowing down through inclined parallel channel and (ii) the system was under the influence of magnetic field. They analysed the effect of magnetic field on the velocity and temperature of the fluid.

In all the above studies¸ the authors have considered a very small magnetic Reynolds number thereby neglected magnetic induction effects in order to make the mathematical analysis of the problem as simple. A study on hydromagnetic free convective flow has been presented by Ghosh, Beg and Zuesco [5] by taking into account the effect of induced magnetic field. Orhan and Ahmet [7] examined the radiation effects on the oscillatory fluid flow of an absorbing or emitting gray gas with induced magnetic field and the solutions were obtained by perturbation method. The numerical study of the hydromagnetic free convective flow in the presence of an induced magnetic field has been performed by Sarveshanand and Singh [8]. In most of these studies only the hydromagnetic flow of an electrically conducting and viscous incompressible fluid are considered. Thus, in this paper, the non-Newtonian MHD flow of a third-grade fluid through a parallel porous channel with induced magnetic field and viscous dissipation are considered. The governing equation which includes momentum, magnetic induction and energy equation have been solved analytically by perturbation method. Further the expression for the induced current density and skin friction are obtained. The effect of various physical parameters on the velocity, the induced magnetic field, the temperature, the induced current density profile is shown graphically while that of the skin friction are tabulated.

Description of the Problem

We consider a steady MHD flow of a third-grade fluid through an infinite porous channel. The upper and lower channel are at $y' = h$ and $y' = -h$ respectively in cartesian coordinate system with the x' - axis parallel to the direction of the flow. This problem is modelled by confine the study to the region $0 \le y' \le h$ and considers the flow to be symmetric about the centreline (i.e $y' = 0$) of the channel.In driven the flow, a uniform magnetic field of strength H_0 is applied perpendicular to the channel along the transverse direction with the consideration of both magnetic Reynolds number and induced magnetic field H'_x. There is cross flow due to a uniform injection of the fluid at one channel which equal to constant suction of velocity v_0 at the other channel. It is assumed that the channel is maintained at a constant temperature T'_ω higher than the ambient temperature T'_∞. The effect of buoyant forces and gravity are neglected. With the above assumptions, the governing equations of the flow are given by

$$\frac{dv'}{dy'} = 0 \text{ which implies that } v' = v_0 \tag{1}$$

$$\frac{dH'_y}{dy'} = 0 \text{ which has a solution } H'_y = H_0 \tag{2}$$

where v_0 and H_0 are constants

$$\mu \frac{d^2u'}{dy'^2} + v'\frac{du'}{dy'} + 6\beta_3\left(\frac{du'}{dy'}\right)^2 \frac{d^2u'}{dy'^2} + \mu_\varepsilon \frac{H_0}{\rho}\frac{dH'_x}{dy'} = 0 \tag{3}$$

$$\frac{1}{\sigma\mu_\varepsilon}\frac{d^2H'_x}{dy'^2} + H_0\frac{du'}{dy'} + v'\frac{dH'_x}{dy'} == 0 \tag{4}$$

$$K\frac{d^2T'}{dy'^2} + \rho c_p v'\frac{dT'}{dy'} + \mu\left(\frac{du'}{dy'}\right)^2 = 0 \tag{5}$$

Subject to the boundary conditions

$$y' = 0: \quad u' = 0,\ H'_x = 0,\ T' = T'_w$$
$$y' \to \infty: \quad u' \to U_0,\ H'_x \to 0,\ T' \to T'_\infty \tag{6}$$

where c_p is the specific heat capacity, K denote thermal conductivity, ρ is the density of the fluid, β_3 is the third grade fluid parameter.

Using the following non-dimensional parameters

$$v_0 = \frac{h\rho v}{\mu},\quad \beta = \frac{6\beta_3}{h^2\mu},\qquad M = \frac{H_0 h}{\mu}\sqrt{\frac{\mu_\varepsilon}{\rho}},\quad B = \sqrt{\frac{\mu_\varepsilon}{\rho U_0}}H'_x,$$

$$P_M = \sigma\mu\mu_\varepsilon,\ \Pr = \frac{\mu\rho c_p}{K},\quad Br = \frac{\mu\rho}{K(T_w - T_\infty)}$$

The governing equation in non-dimensional form have taken the form

$$\frac{d^2u}{dy^2}+v_0\frac{du}{dy}+\beta\left(\frac{du}{dy}\right)^2\frac{d^2u}{dy^2}+M\frac{dB}{dy}=0 \tag{7}$$

$$\frac{d^2B}{dy^2}+v_0P_M\frac{dB}{dy}+MP_M\frac{du}{dy}=0 \tag{8}$$

$$\frac{d^2T}{dy^2}+v_0P_r\frac{du}{dy}+B_r\left(\frac{du}{dy}\right)^2=0 \tag{9}$$

with the corresponding boundary conditions in non-dimensional form as

$$y=0: \qquad u=0, \quad B=0, \; T=1 \tag{10}$$

$$y=1: \qquad u=1, \quad B=0, \; T=0 \tag{11}$$

Method of Solution

Equation (7) – (9) with boundary condition (10) and (11) are solved by taken $\beta=\varepsilon$ as a small parameter

and apply the perturbation method.

$$u(y)=u_0(y)+\varepsilon u_1(y)+O\left(\varepsilon^2\right) \tag{12}$$

$$B(y)=B_0(y)+\varepsilon B_1(y)+O\left(\varepsilon^2\right) \tag{13}$$

$$T(y)=T_0(y)+\varepsilon T_1(y)+O\left(\varepsilon^2\right) \tag{14}$$

Substituting equation (12) – (14) into equation (7) - (11) and collect the lie terms base on the power of ε and neglecting term of $+O\left(\varepsilon^2\right)$, the following ordinary differential equations are obtained

$$\varepsilon^0: \qquad \frac{d^2u_0}{dy^2}+v_0\frac{du_0}{dy}+M\frac{dB_0}{dy}=0 \tag{15}$$

$$\frac{d^2B_0}{dy^2}+v_0P_M\frac{dB_0}{dy}+MP_M\frac{du_0}{dy}=0$$

(16)

$$\frac{d^2T_0}{dy^2}+v_0P_r\frac{dT_0}{dy}+B_r\left(\frac{du_0}{dy}\right)^2=0$$
(17)

$$y=0: \qquad u_0=0,\ B_0=0, \qquad T_0=1 \tag{18}$$

$$y=1: \qquad u_0=1,\ B_0=0, \qquad T_0=0 \tag{19}$$

$$\varepsilon\ \vdots \qquad \frac{d^2u_1}{dy^2}+v_0\frac{du_1}{dy}+6\left(\frac{du_0}{dy}\right)^2\frac{d^2u_0}{dy^2}+M\frac{dB_1}{dy}=0 \tag{20}$$

$$\frac{d^2B_1}{dy^2}+v_0P_M\frac{dB_1}{dy}+MP_M\frac{du_1}{dy}=0$$
(21)

$$\frac{d^2T_1}{dy^2}+v_0P_r\frac{dT_1}{dy}+2B_r\frac{du_0}{dy}\frac{du_1}{dy}=0$$
(22)

$$y=0: \qquad u_1=0,\ B_1=0, T_1=0 \tag{23}$$

$$y=1: \qquad u_1=0,\ B_1=0, \qquad T_1=0 \tag{24}$$

Equations (15)-(17) and (20)-(22) with boundary conditions (18), (19) and (23), (24) respectively are coupled system of ordinary differential equation (ODE) with constant coefficient. These systems of ODE are separately solved analytically by theory of simultaneous ODE and the solution are given as follows

$$u_0=a_8c_2+a_9c_1+a_6c_3\exp(a_1y)+a_7c_4\exp(a_2y) \tag{25}$$

$$B_0=c_2+c_3\exp(a_1y)+c_4\exp(a_2y) \tag{26}$$

$$T_0=c_6-c_5a^{-17}\exp(-a_{17}y)-a_{18}\exp(a_1+a_2)y-a_{19}\exp(2a_1y)-a_{20}\exp(2a_2y) \tag{27}$$

$$\begin{aligned}u_1=&\ a_{60}c_7+a_{54}c_{10}+a_{61}\exp(3a_2y)+a_{62}c_9\exp(a_{25}y)+a_{63}c_8\exp(a_{26}y)+a_{64}\exp(a_{45}y)\\&+a_{65}\exp(a_{44}y)\end{aligned} \tag{28}$$

$$\begin{aligned}B_1=&\ c_{10}+a_{39}c_9\exp(a_{25}y)+a_{40}c_8\exp(a_{26}y)+a_{41}\exp(a_1y)(\cosh(a_2y)+\sinh(a_2y))^2\\&+a_{42}\exp(a_2y)(\cosh(a_1y)+\sinh(a_1y))^5+a_{43}(\cosh(a_2y)+\sinh(a_2y))^3\end{aligned} \tag{29}$$

$$
\begin{aligned}
T_1 = c_{12} &+ c_{11}a_{106}\exp(a_{17}y) + a_{107}\exp(a_{76}y) + a_{108}\exp(a_{77}y) + a_{109}\exp(a_{78}y) \\
&+ a_{110}\exp(a_{79}y) + a_{111}\exp(a_{80}y) + a_{112}\exp(a_{81}y) + a_{113}\exp(a_{82}y) + a_{114}\exp(a_{83}y) \\
&+ a_{115}\exp(a_{84}y) + a_{116}\exp(a_{85}y)
\end{aligned}
\tag{30}
$$

Equations (25) - (30) are substituted into equations (12) – (14) to obtain the solution for $u(y), B(y)$ and $T(y)$ which are presented in form of graphs. The induced current density is given by

$$
\begin{aligned}
J = -\frac{dB}{dy} &= a_{117}\exp(5a_1 + a_2)y + a_{118}\exp(a_1 + 2a_2)y - c_3a_1\exp(a_1y) - c_4a_2\exp(a_2y) \\
&- \varepsilon(c_9a_{39}a_{25}\exp(a_{25}y) + c_8a_{40}a_{26}\exp(a_{26}y) + 3a_{43}a_2\exp(3a_2y))
\end{aligned}
\tag{31}
$$

The parameters $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_{118}$ and $c_1, c_2, c_3, \ldots\ldots\ldots\ldots, c_{12}$ are defined in the appendix.

$$
\tau_0 = \left(\frac{du}{dy}\right)_{y=0} = a_6c_3a_1 + a_7c_4a_2 + \varepsilon(a_{25}a_{62}c_9 + a_{26}a_{63}c_8 + 3a_2a_{61} + a_{44}a_{65} + a_{45}a_{64})
\tag{32}
$$

$$
\begin{aligned}
\tau_1 = -\left(\frac{du}{dy}\right)_{y=1} &= -a_6c_3\exp(a_1)a_1 - a_7c_4\exp(a_2)a_2 - \varepsilon(a_{61}(3\exp(3a_2)a_2) + a_{62}c_9\exp a_{25} \\
&- a_{63}c_8\exp(a_{26})a_{26} + a_{64}\exp(a_{45})a_{45} + a_{65}\exp(a_{44})a_{44}
\end{aligned}
\tag{33}
$$

Results and Discussion

In this section, the investigation of non-Newtonian MHD flow of a third-grade fluid in a porous channel under the influence of an induced magnetic field and viscous dissipation are discussed. The computed results in form of graphs and tables of velocity profile $u(y)$, induced magnetic field $B(y)$, temperature profile $T(y)$ and skin friction τ_0, τ_1 respectively are produced for various governing flow parameters. The effect of the suction parameter (v_0), magnetic field parameter (M), induced magnetic parameter (P_M) and Prandtl number (P_r) on flow are analysed.

Figure 1 - 3 show the effect of various values of v_0, M and P_M on velocity distribution respectively. Figure 1 shows the effect of v_0 on velocity profile when $M = 0.5,\ P_M = 0.5$ and $\beta = 0.001$. It is notice that the velocity increases with increase in v_0 due to the suction at lower channel. Figure 2 presents velocity profile due to the variation of magnetic field parameter M when $v_0 = 1.0,\ P_M = 0.5$ and $\beta = 0.001$ at various cross-sections of the channel. It is observed that $u(y)$ increases as M increases due to the suction at lower channel and later decreases due to increasing of magnetic damping force on

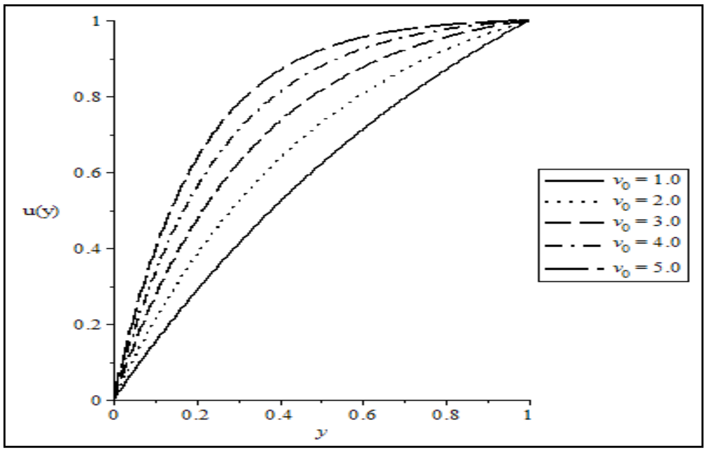

Figure 1 Velocity profile $u(y)$ for different values of v_0

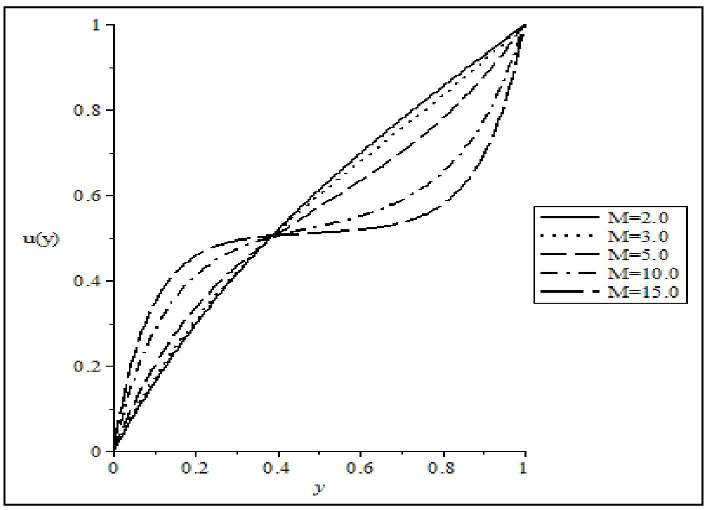

Figure 2 Velocity profile $u(y)$ for different values of M

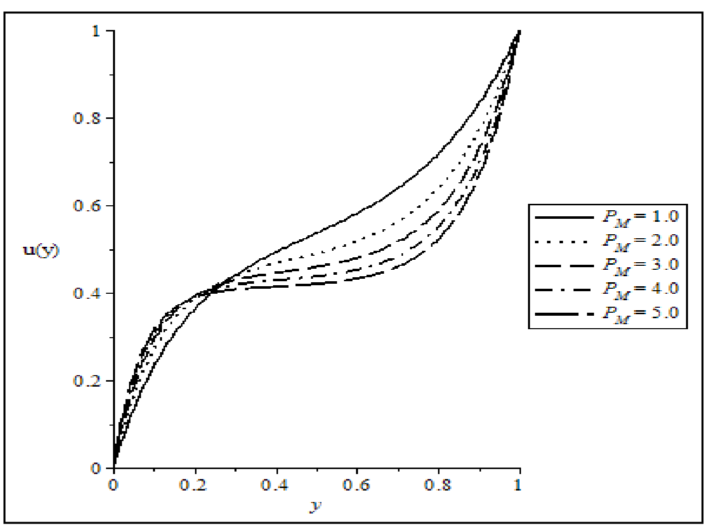

Figure 3 Velocity profile $u(y)$ for different values of P_M

$u(y)$ at upper channel. This is illustrated by the crossing of the curve $u(y)$ for various values of M .The effect of magnetic Prandtl number P_M on velocity profile $u(y)$ are presented in Figure 3 when $v_0 = 1.0,\ M = 0.5$ and $\beta = 0.001$. It is noticed that $u(y)$ slightly accelerate as P_M increases and later reduce slightly towards the upper channel. Here, the slightness increments and decrements are due to a weak magnetic field imposed on $u(y)$.

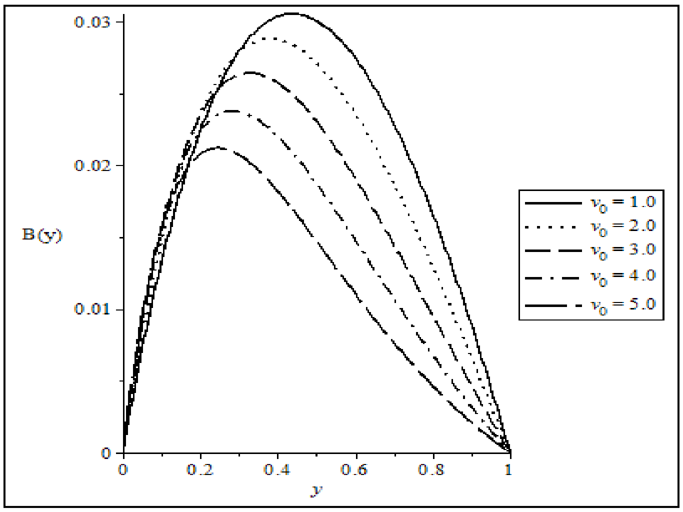

Figure 4 Induced magnetic field $B(y)$ for different values of v_0

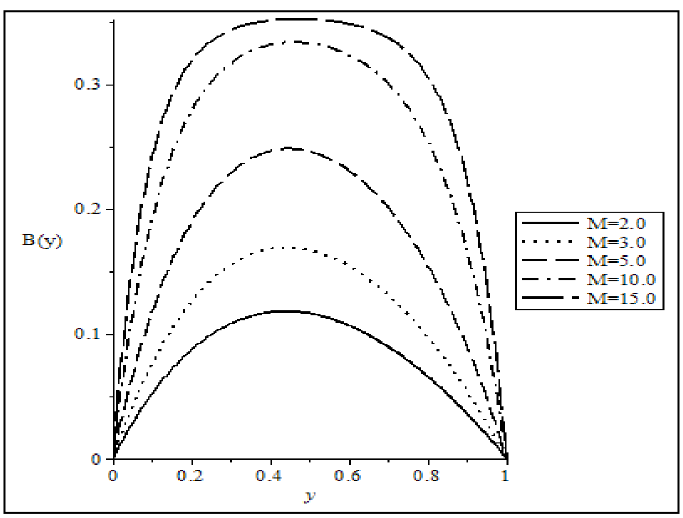

Figure 5 Induced magnetic field $B(y)$ for different values of M

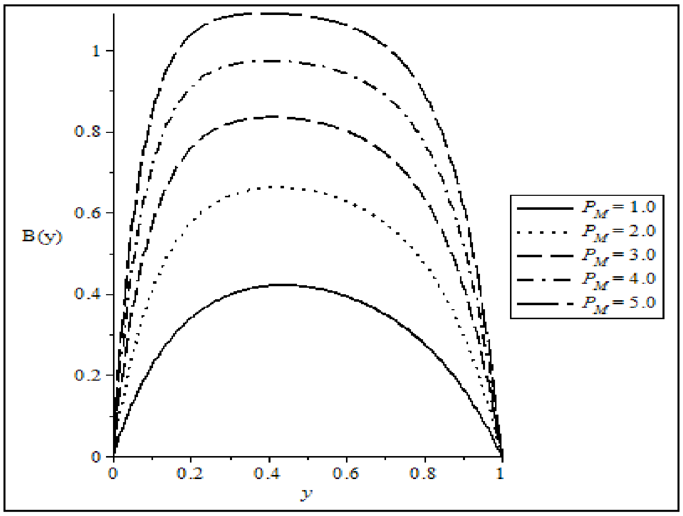

Figure 6 Induced magnetic field $B(y)$ for different values of P_M

Figure 4 - 6 describe the effect of various values of v_0, M and P_M on induced magnetic field respectively.

Figure 4 shows the distribution of the induced magnetic field $B(y)$ with the suction parameter v_0 for $M = 0.5,\ P_M = 0.5$ and $\beta = 0.001$. Initially, it is noticed that $B(y)$ are asymmetric about $y = 0$ because of the suction at the lower channel but as v_0 increases, $B(y)$ decrease due to the rising in induced magnetic flux for all distances into the boundary layer, transverse to the channel. Figure 5 and 6 depicts $B(y)$ with M when $v_0 = 1.0,\ P_M = 0.5$, $\beta = 0.001$ and P_M when $v_0 = 1.0,\ M = 5.0$, $\beta = 0.001$ respectively. It is observed the $B(y)$ increases when M and P_M increases. This is cause by induced magnetic field which produces its own magnetic field in the fluid resulting to modification of original magnetic field. The flow in magnetic field generates a mechanical force which modifies the motion of the fluid.

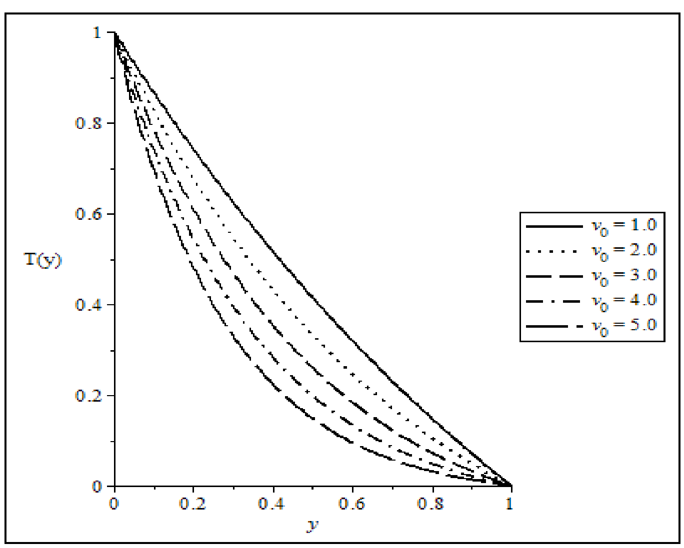

Figure 7 Temperature profile $T(y)$ for different values of v_0

Figure 7 and 8 illustrates the effect of various values of v_0 and P_r on temperature profile respectively. In figure 7, it is seen that, temperature profile $T(y)$ decreases with increase in suction parameter v_0 due to due to the convection of the fluid from regions in the lower half to centre which has higher fluid speed. Similarly, from figure 8, it is noticed that the increases P_r causes decreases in $T(y)$. This is because of decreasing thermal boundary layer thickness which reduces the average temperature within the boundary layer.

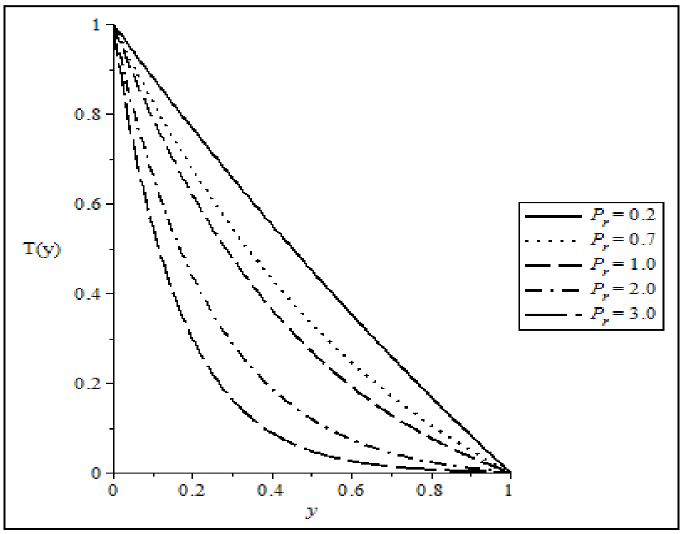

Figure 8 Temperature profile $T(y)$ for different values of P_r

The effects of v_0, M and P_M on the skin friction on the channels are shown in the Table 1. This table clearly shows the skin friction on both part of the channel increases with increase in v_0, although negatively increases on the channel $y=1$. Similarly, it is notice that as M and P_M increases, the skin friction increases at $y=0$ and decreases negatively at $y=1$

Table 1 Effect of suction parameter v_0, magnetic field parameter M and magnetic Prandtl number P_M on skin friction

v_0	$M=0.5, P_M=0.5, P_r=0.7,$ $Br=0.007$ and $\varepsilon=0.001$		M	$v_0=1.0, P_M=0.5, P_r=0.7,$ $Br=0.007$ and $\varepsilon=0.001$		P_M	$v_0=1.0, M=5.0, P_r=0.7,$ $Br=0.007$ and $\varepsilon=0.001$	
	τ_0	τ_1		τ_0	τ_1		τ_0	τ_1
1.0	1.590527	-0.592025	2.0	1.731470	-0.733567	1.0	3.034699	-2.046634
2.0	2.316491	-0.321501	3.0	1.908768	-0.911747	2.0	3.981020	-3.007410
3.0	3.150922	-0.163653	5.0	2.409879	-1.416097	3.0	4.711782	-3.755229
4.0	4.052004	-0.079275	10.0	4.032554	-3.064446	4.0	5.319343	-4.381640
5.0	4.985849	-0.037170	15.0	5.730419	-4.842651	5.0	5.844976	-4.927491

Conclusions

The study considered the MHD flow of a third-grade fluid through an infinite porous channel under the influence of external magnetic field by taking into account the effect of induced magnetic field and viscous dissipation. The effect of the various physical parameters obtained on velocity, induced magnetic, and temperature field as well as skin friction are presented in form graphs and table.
The presents results obtained are listed below

1. Velocity of the fluid increases with increasing suction parameter and later decreases as magnetic parameter and magnetic Prandtl number increases. This is due to the fact that the magnetic and induced magnetic field has apparent effect than suction velocity.
2. Induced magnetic field decreases with increase in suction parameter due to rising in magnetic flux.
3. An increasing magnetic parameter and magnetic Prandtl number increases the induced magnetic field as a result of magnetic field in the fluid produced by induced magnetic field itself.
4. Temperature of the fluid decreases with an increase in suction parameter due to the influence of convection in pumping the fluid from the lower region to the centre of the channel. Similarly, the temperature field decreases with increase in Prandtl number because of increasing in thermal conductivities which diffuses heat away from the heated channel.
5. Skin friction increases and decreases at lower and upper part of the channel respectively as magnetic parameter and magnetic Prandtl number increases.

Recommendations

The study recommended that the velocity of the fluid and induced magnetic field can be influence by modify the suction / injection velocity on the porous channel while making engineering design. Further study on this research can be on unsteady flow and numerical method for the solution of the equations governed the flow.

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