Simulation of Mathematical Model for Ecological Surveillance of Prey-Predator

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Abstract
Mathematics is integral to the study of biological systems. Mathematical models can help researchers explain natural phenomena quantitatively and generate new hypotheses better than with only experimental observations. After translating a biological problem into a collection of mathematical equations, solutions are sought-after and visualized. One of such models is the Lotka-Volterra, which is a representation that models the population of two species. One species is a predator and the other is the prey. The aim of this research work is to design and simulate a computational ecological surveillance system with specific objective to provide an improved numerical solution of the ecological Lotka-Volterra predation model using the mathematical eigen-value and vector method approach. Results of the numerical solutions are presented to show the behaviour of the model and was also favourable with other existing models. It is known that prey-predator activities are arguably building blocks of the bio and ecosystems as biomasses are grown out of their resource masses. Species contend, evolve and disperse merely for the aim of seeking resources to sustain their struggle for their terribly existence. The ecological surveillance system is implemented to show visual interactions between predator and preys which may be used in fields of forestry to aid conservationist and safe countless animals from extinctions. It can also be used in farms to monitor infiltration of predators on crops.

Keywords: Predation; Mathematical Modelling; Ecology; Population.

Introduction

Ecology or environmental biology is the study of the relationships between organisms and their environment. The concept of environment includes other organisms and physical surroundings. It involves relationships between people within a population and between people of various populations. These interactions between individuals, between populations, and between organisms and their environment form ecological systems, or ecosystems. Ecology has been outlined variously as "the study of the interrelationships of organisms with their surroundings and each other," as "the economy of nature," and as "the biology of ecosystems" (Robert and Stuart, 2019).

Roth (2016) says that all animals are either predators or prey and in most cases, are both; also such interactions involved in attempting to eat and avoid being eaten have strong and wide-reaching influences across all facets of ecology, from behavioural, population, and community interactions to how we attempt to manage and conserve the natural world.

A biological environment consisting of rabbits and foxes living together. Foxes eat the rabbits and rabbits eat clover. Suppose that there are enough clovers and the rabbits have enough food to eat. When there are a lot of rabbits, the foxes also grow and their population increase. When the number of foxes increases and they eat a lot of rabbits, they enter into a short period of food and their number decrease. As the range of the foxes decreases, the rabbits will be safe and their population increase. When the number of rabbits increases the number of foxes would increase and bypassing the time, we can see infinite repeatability of increase and decrease in the population of these two kinds of animals. (Biazar and Montazeri, 2017). This illustration shows that the prey and predator evolve together. The prey is a component of the predator's setting, and the predator dies if it does not get food, so it evolves whatever is necessary in order to eat the prey: speed, stealth, camouflage (to hide while approaching the prey), a good sense of smell, sight, or hearing (to find the prey). Likewise, the predator is a component of the prey's setting, and the prey dies if it is eaten by the predator, so it evolves whatever is necessary to avoid being eaten: speed, camouflage (to hide from the predator), a good sense of smell, sight, or hearing (to detect the predator), thorns, poison (to spray when approached or bitten) and so on (Taleb, 2016).

To study the association or interaction in predation, various predator-prey models exist to describe the association between various preys and predators, one of such is the fundamental ecological model called Lotka-Volterra (by Alfred Lotka and Vitto Volterra) predator model. This model primarily focuses on a predator to a prey and was described by Vitto from studying the increase and fall of sea fishing fleets. When fishing was smart, the number of fishermen increased, drawn by the success of others. After a time, the fish declined, perhaps due to over-harvest, and then the number of fishermen also declined. After some time, the cycle repeated. A main purpose of modelling population interactions is to understand what causes such fluctuations. Indeed, the very first Lotka-Volterra system is the result of such an effort. (Eugeny *et al.*, 2016).

Lotka-Volterra Predation Model

Differential equations are extremely useful in modelling various biological phenomena. One of such examples is the combination of Lotka-Volterra equations, which is a representation that models the population of two species. One species is a predator and the other is the prey. The Lotka-Volterra equations is one of the oldest predator-prey models. It was proposed by the Biophysicist Alfred Lotka in 1910 to model chemical reactions, and was modified by the Mathematician Vito Volterra in 1925 when he was attempting to explain the oscillating fish populations, he discovered in the Mediterranean. However, the Lotka-Volterra equations can be applied to many other species as well. The Hudson Bay Company (a famous Canadian fur trading company) for example observed a similar oscillatory behaviour in the populations of the predatory lynx and its prey, the hare, back in 1840 (Anderson and Blake, 2012).

The Lotka-Volterra Equations is a system of first-order, nonlinear ODEs. The simplest form ofthe system is the following model:

$$\frac{du}{dt} = c_1 u - a_{12} uv \quad (1)$$

$$\frac{dv}{dt} = -c_2 v + a_{21} uv \quad (2)$$

In these equations u and v represents prey and predator populations square measure is shown correspondingly,c_1 is the specific rate of prey population growth in predator's absence (i.e the reproduction rate of the prey), a_{12} is a constant, characterizing the rate of predators' consumption of prey species (death rate of the prey due to the presence of the predator). The greater a_{12} is, the greater the death rate of the prey is due to predation and the more effective the predator is killing the prey), c_2 is the specific rate of predator's mortality (death rate of the predator. The greater c_2 is, the greater the death rate of the predator is. a_{21} is a constant that characterizes the rate of multiplied number of predators because of preys' death (reproduction rate of the predator). The greater a_{21} is, the more rapidly the predator reproduces and the more effectively the prey is able tonourish the predator.

Lotka-Volterra predation equation is based on the following assumptions:

1. Predator can eat limitless;
2. Supply of food resource (i.e. prey) depends on the prey population size;
3. The rate of change of population directly depends on its size;
4. Environment is constant, inconsequential genetic adaptations for both species;
5. All time unlimited food supply for the prey.

The equation is continuous and deterministic indicate continuous overlapping of prey and predator population (Eugeny *et al.*, 2016).

Numerical Solution

The Lotka-Volterra Equations is a system of first-order, nonlinear ordinary differential equations (ODE). The simplest form ofthe system is the following model:

$$\frac{du}{dt} = c_1 u - a_{12}uv \quad (3)$$

$$\frac{dv}{dt} = -c_2 v + a_{21}uv \quad (4)$$

Let the co-efficient matrix be represented as

$$x' = \begin{pmatrix} c_1 & -a_{12} \\ -c_2 & a_{21} \end{pmatrix}(x) \quad (5)$$

We find the eigenvalues:

$$\begin{vmatrix} c_1 - \lambda & -a_{12} \\ -c_2 & a_{21} - \lambda \end{vmatrix} = 0$$

Therefore,

$$\lambda^2 - (c_1 + a_{21})\lambda + (c_1 a_{21} - c_2 a_{12}) = 0 \quad (6)$$

Solving equation (6) quadratically with respect to λ, we obtained

$$\lambda = \frac{1}{2}\left[c_1 + a_{21} \pm \sqrt{{c_1}^2 + 2c_1 a_{21} + {a_{21}}^2 - 4c_1 a_{21} + 4c_2 a_{12}}\right] \quad (7)$$

if L = $\sqrt{{c_1}^2 + {a_{21}}^2 - 2c_1 a_{21} + 4c_2 a_{12}}$ (8)

Then

$$\lambda = \frac{1}{2}[c_1 + a_{21} \pm \mathrm{L}] \quad (9)$$

and therefore,

$$\lambda_1 = \frac{1}{2}[c_1 + a_{21} + \mathrm{L}],\ \lambda_2 = \frac{1}{2}[c_1 + a_{21} - \mathrm{L}] \quad (10)$$

To find the eigenvector corresponding to λ_1, we solve:

Let $\begin{vmatrix} c_1 - \lambda_1 & -a_{12} \\ -c_2 & a_{21} - \lambda_1 \end{vmatrix} \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = 0$ (11) i.e. $(c_1 - \lambda_1)k_1 - a_{12}k_2 = 0$ (12)

$$-c_2 k_1 + (a_{21} - \lambda_1)k_2 = 0 \quad (13)$$

Solving equation (12) & (13) simultaneously to obtain k_1 & k_2

$k_1 = a_{12}, k_2 = c_1 - \lambda_1$ or $k_1 = a_{21} - \lambda_1, k_2 = c_2$

Similarly, the eigenvector corresponding to λ_2 can easily be obtained:

$$\begin{vmatrix} c_1 - \lambda_2 & -a_{12} \\ -c_2 & a_{21} - \lambda_2 \end{vmatrix} \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = 0 \quad (14)$$

$k_1^* = a_{12}, k_2^* = c_1 - \lambda_2$

$k_1^* = a_{21} - \lambda_2, k_2^* = c_2$

$$\mathrm{x} = \begin{bmatrix} \mathrm{u} \\ \mathrm{v} \end{bmatrix} = \mathrm{d}_1 \begin{bmatrix} \mathrm{k}_1 \\ \mathrm{k}_2 \end{bmatrix} \mathrm{e}^{\lambda_1^{\mathrm{t}}} + \mathrm{d}_2 \begin{bmatrix} k_1^* \\ k_2^* \end{bmatrix} \mathrm{e}^{\lambda_2^{\mathrm{t}}} \quad (15)$$

From the value of x above, the new equation is formulated thus:

$$u = \mathrm{d}_1\mathrm{k}_1\mathrm{e}^{\lambda_1^{\mathrm{t}}} + \mathrm{d}_2 k_1^* \mathrm{e}^{\lambda_2^{\mathrm{t}}} \quad (16)$$

$$v = \mathrm{d}_1\mathrm{k}_2\mathrm{e}^{\lambda_1^{\mathrm{t}}} + \mathrm{d}_2 k_2^* \mathrm{e}^{\lambda_2^{\mathrm{t}}} \quad (17)$$

Existence of Solution

From equ. (8)

$$\mathrm{L}^2 = {c_1}^2 + {a_{21}}^2 + 4c_2 a_{12} - 2c_1 a_{21} \quad (18)$$

For L to be a real number, then ${c_1}^2 + {a_{21}}^2 + 4c_2 a_{12} > 2c_1 a_{21}$ (19)

The inputs of the formulated model are summarized below, where:

d_1 & d_2 are positive arbitrary constants;

$k_1 = (a_{21} - \lambda_1)$ or a_{12} (20)

$k_1^* = (a_{21} - \lambda_2)$ or a_{12} (21)

$k_2 = (c_1 - \lambda_1)$ or c_2; (22)

$k_2^* = (c_1 - \lambda_2)$ or c_2; (23)

$\lambda_1 = \frac{1}{2}[c_1 + a_{21} + L]$ (24)

$\lambda_2 = \frac{1}{2}[c_1 + a_{21} - L]$ (25)

$L = \sqrt{c_1{}^2 + a_{21}{}^2 - 2c_1 a_{21} + 4c_2 a_{12}}$ (26)

t = time interval

Numerical Simulation

All variables of the formulated equations are derived from the inputs of the Lotka-volterra equations such as c_1, c_2, a_{21}, a_{12}. Several replications or runs of the newly formulated model were done in Microsoft Excel worksheet to show graphical representation using some tested parameters, all these are represented in tables and figures. The Figure 1 shows the result gotten in graph of prey and predator with input given as:

c_1= 0.2, c_2= 0.5, a_{21}= 0.5, a_{12}= 0.2, t = 20 units

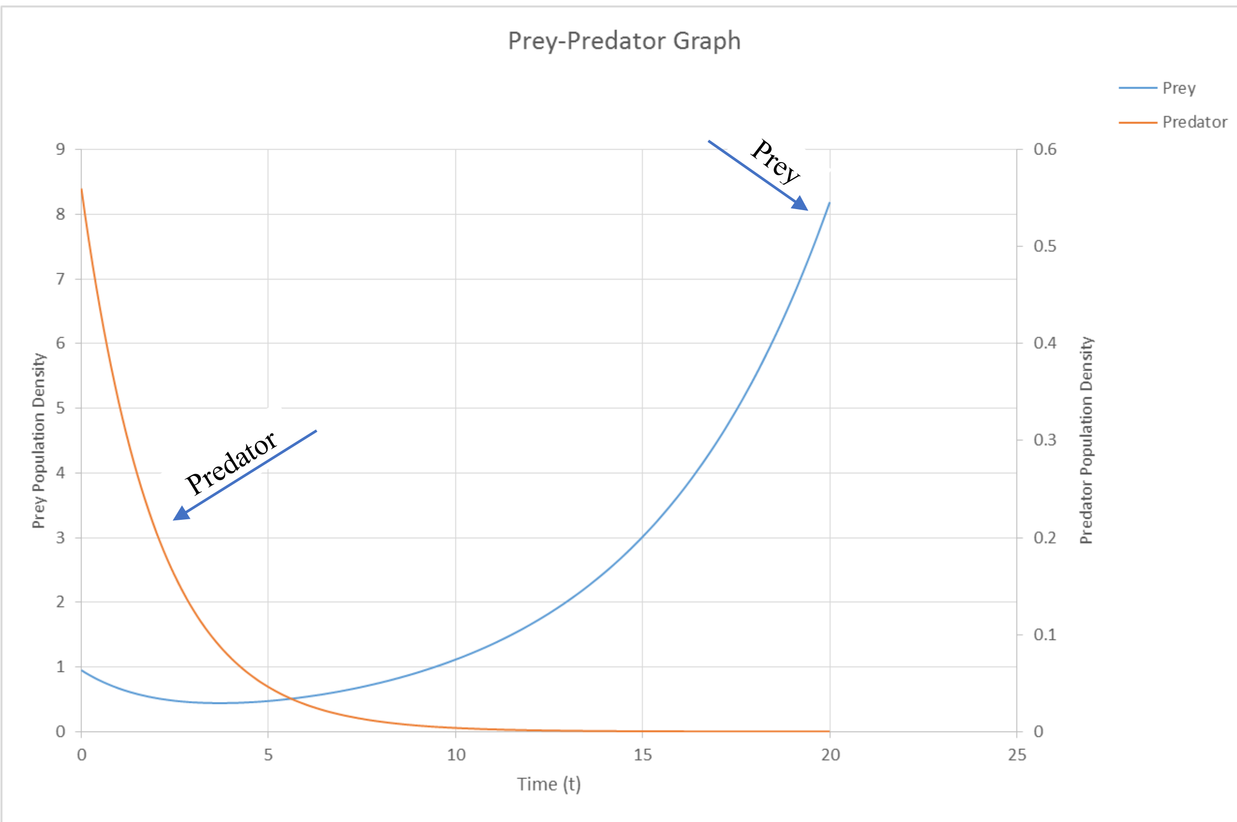

Figure 1 Prey-Predator Graphical Representation

The graph shows the onset of the animal community, the predator-population is greatly increased against the prey-population and gradually declines as a result of insufficient food (i.e. very low amount of starting prey population) which could cause death as a result of starvation, migration, chemical or biological/man control and ageing. The prey-population in turn increases gradually in response to the decrease in the predator-population and reaches its peak population as the predator population declines to zero and finds it impossible to bounce back in respect to a natural phenomenon.

Ecological Surveillance Design

After an effective and efficient design of the prey-predator system as shown, figure 2 represents appropriate experimental design and condition for simulation runs (i.e. the surveillance system is designed such that it will only take positive inputs as parameters for successful run). Figure 3 shows the activity diagram of animal life cycle. This will in turn lead to the production of system specification otherwise regarded as the system documentation which consists of logical steps to be taken for the actualization of the proposed system. The program written is object oriented based which immensely contributing to the attainment of the desired result design. The desired integrated development environment for the ecological surveillance was implemented in the Netbeans (a popular IDE for Java).

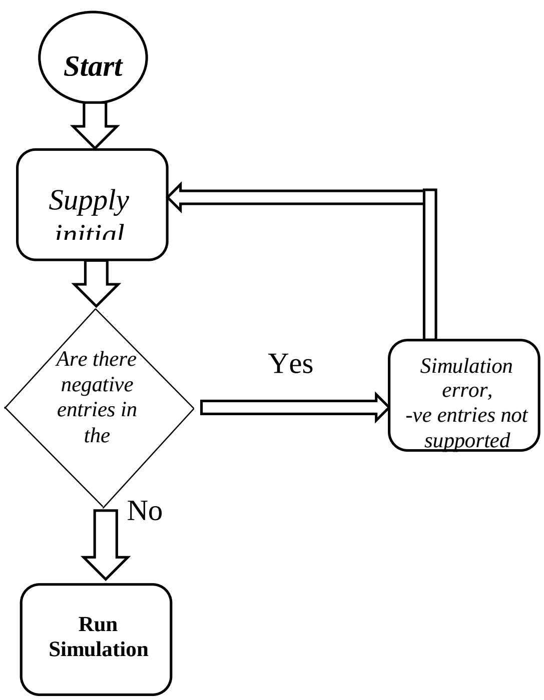

Figure 2: Ecological Simulator Design

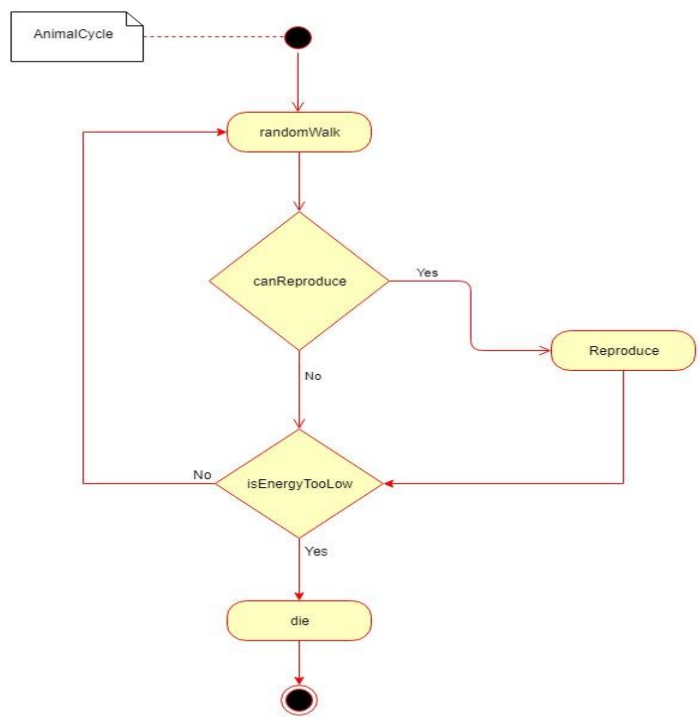

Figure 3: Typical Design for an animal life-cycle

Results and Discussions

Ecological Surveillance: Simulation 1 [Prey = 50, Predator = 0]

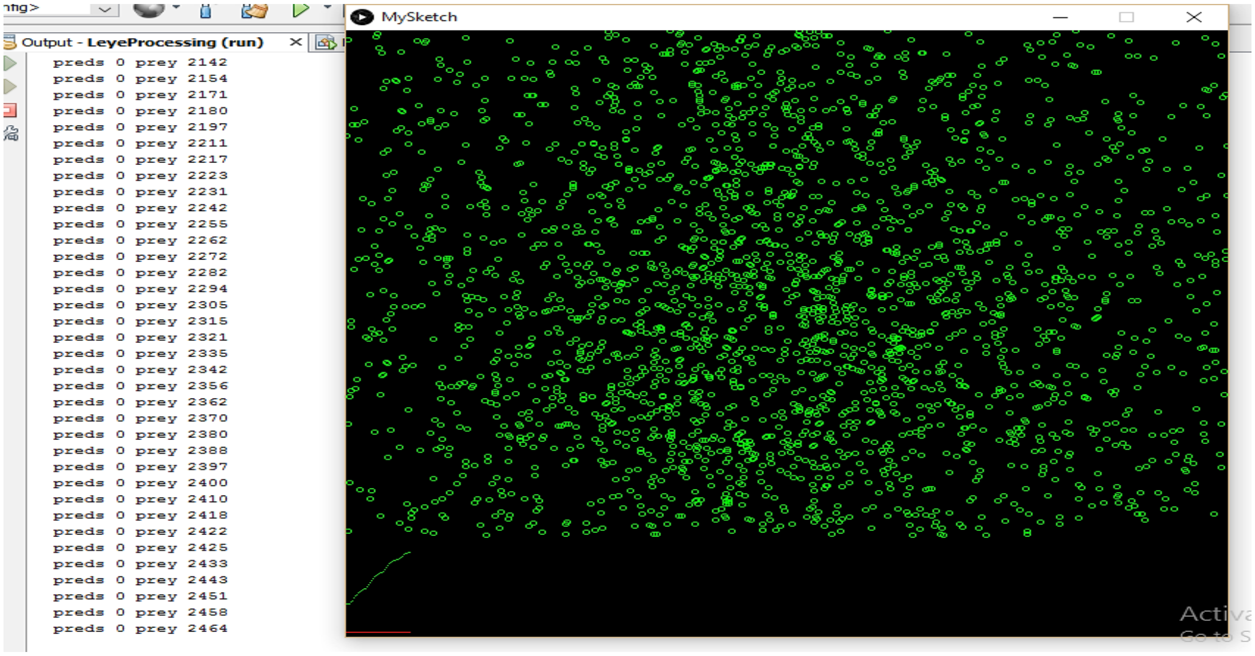

Figure 4: Simulation 1: Prey (50), Predator (0)

The first simulation run (Figure 4) simulates a prey population in a predator free habitat (i.e preys = 50, Predator = 0). The preys without disturbance feed freely, reproduce and increase rapidly in number while a few dies due to natural phenomenon (sickness, ageing) only.

Ecological Surveillance: Simulation 2 [Prey = 200, Predator = 10]

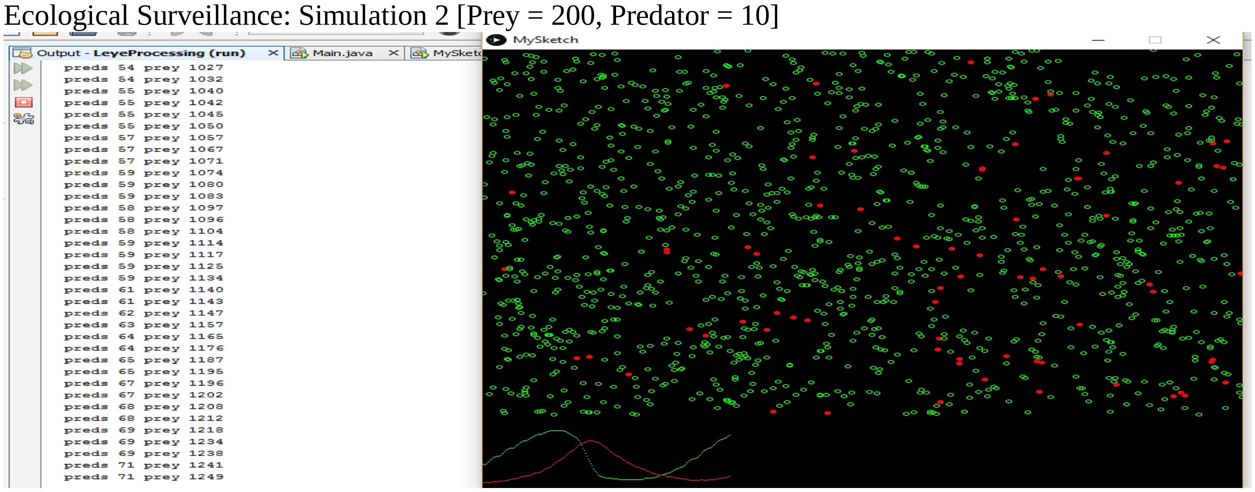

Figure 5: Simulation 2: Prey (200), Predator (10)

The second simulation run in Figure 5 introduces an initial population 10 predators (predators are represented in red colour) in an environment consisting of population of 200 preys (preys are represented in green colour). The assumptions follow the Lotka-Volterra predation model that the predators feed limitless and feed only on one kind of prey. This introduction ten predators causes a stir in the prey habitat as the predators competes with the prey and hunts the prey for food. This causes a significant drop in the prey population due to increase in preys' mortality. On the other hand, the predator population increases rapidly to pose more threat to the prey population. At this point, the prey population decreases while the predator population increases, and the cycle continues until the ecological surveillance eventually terminates the system at the point when the available predators pose great effect on the prey population (when prey population is likely to go into extinction), and suggests that some predator control techniques should be applied to avoid the extinction of the animal (prey).

Conclusions

It is noteworthy that the ratio of the organisms over time (especially in the simulations) strongly resembles the Lotka-Volterra equations which are used to describe such biological systems. This variation produces interesting population dynamics, but is not usually stable. Thus, increase in prey population implies increase in predator population; increase in predator population implies decrease in prey population; decrease in prey population implies decrease in predator population; and finally decrease in predator population implies increase in prey population.

Recommendations

The results of present analysis may be useful in the field of forestry, agriculture and fishery. Conservationist have saved countless rare animals from extinction, the ecological surveillance could be also used in farms to monitor infiltration of predators on crops, which will help to lead to an increase in yield of crops. Since scope of this work considers a single-predator to a kind of prey, further research could be carried out on a predator feeding on multiple kinds of prey, or multiple predators to multiple kinds of preys and vice-versa.

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