Analytical Solution of a Reactive Hydromagnetic Fluid Through Porous Media Between Permeable Pipes Under Optically Thick Limit Radiation

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Abstract
In this paper, we investigated the flow of a reactive hydromagnetic fluid through porous media between permeable beds under optically thick limit radiation. The Navier-Stokes equations employed for the flow between the permeable beds are presented with the Mathematical modeling for the velocity field and mass flux of the flow using direct method to solve the governing equation in order to get the exact solution with the appropriate boundary conditions.The solution of velocity was used to examine the influence of magnetic strength on a steady state of a reactive hydromagnetic fluid flow under optically thick limit radiation. The results were presented graphically showing the effects of Hartmann number (M), Reynolds number (R) and the Darcy number (Da) against the velocity profile on the flow regime.

Keywords: Hydromagnetic; Fluid; Optically Thick Limit; Porous Media; Pipes.

Introduction

Fluid Mechanics is the study of fluids either at rest (fluids static) or in motion (Fluids dynamics and kinematics). Fluids, unlike solids, lack the ability to offer sustained resistance to a deforming force. Thus, a fluid is a substance which is capable of attaining the shape of its container and retaining that shape at all times in the absence of external forces. Hydromagnetic flow is important due to its industrial applications. For instance, it is used to deal with the problem of cooling of nuclear reactors by fluids having very low Prandtl number. It is used as a coolant because of very high thermal conductivity. It can transport heat even if small temperature difference exists between the surface and the fluid.

The reactive hydromagnetic fluid flow in a porous medium is one of the oldest chapters of the engineering sciences as a result of its rapid growth, widespread industrial and environmental applications. It is still a subject of current research interest and special attention has been paid to the internal flows of a reactive hydromagnetic fluid in ducts and chemicals filled with porous media.

It was observed generally that natural channels have permeable beds and flow resistance coefficient and velocity distributions were derived irrespective of bed porosity. Less study has been undertaken on a reactive hydromagnetic fluid through porous media in a channel or pipe that possesses permeable beds (Prasad and Kumar, 2011).

An early study in this direction was presented by Srinivas and Malathy (2008) studied the pulsatile flow of a hydromagnetic fluid between permeable beds. Nowadays, MHD studies have become a useful tool for researchers due to its wide applications. Makinde (2005) presented the combined effects of a transverse magnetic field and heat transfer on unsteady flow of a conducting fluid through a channel field with saturated porous medium. Attia (2008) investigated the unsteady hydromagnetic couette flow of a dusty fluid with temperature dependent viscosity and thermal conductivity under exponential decaying pressure gradient. Olayiwola (2016) also presented a new computational method for the solution of nonlinear Burgers' equation arising in longitudinal dispersion phenomena in fluid flow through porous media

In this paper, we considered the flow of a reactive hydromagnetic fluid through a porous medium between two permeable beds. An optically thick limit radiation and a uniform magnetic field along with the energy equation in a reaction normal to the flow saturated porous medium between and through permeable beds were considered. The Navier-Stokes equations, that is, the continuity, momentum and the energy equations were employed for the flow between and through the permeable beds respectively. Analytical solutions to the velocity field, momentum and energy equations were obtained. The dimensionless governing equations were solved using direct method by splitting the momentum equation into particular and complimentary functions in order to obtain the exact solution using the appropriate and related boundary conditions. These results are presented in a graphical form for different choices of M, Da^{-1}, n, α, σ, R and t. The approximated solution of the energy equation was obtained using both direct integration and series solution of Adomian Decomposition Method (ADM). The geometry of the problem is shown in figure 1 where L is the characteristics length of the channel.

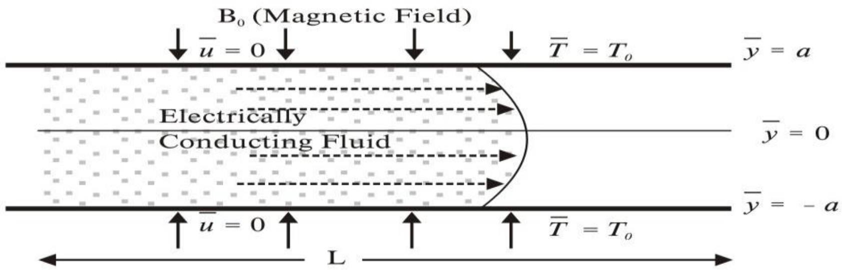

Figure 1: Geometry of the problem

Mathematical Formulation and Basic Solution

Continuity Equation

The continuity equation is simply a mathematical expression of the principle of conservation of mass. The differential form of the continuity equation along *x* –and *y* – directions are given as follows:

$$\nabla . u = 0 \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

The Navier-Stokes equation form a vector continuity equation describing the conservation of linear momentum. If the fluid is an incompressible flow, the flow v is constant and the mass continuity equation simplifies to a volume continuity equation (1) which means that the divergence of velocity field is zero everywhere. The mass of a fluid is conserved.

Momentum Equation

The solution of the Navier-Stokes equations is a flow velocity. It is a field since it is defined at every point in a region of space and an interval of time. Once the velocity field is calculated, other quantities of interest such as pressure and temperature may be found. The momentum equation employed for this research work is written as;

$$\frac{\partial \bar{u}}{\partial t} + V\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{1}{\rho}\frac{\partial p}{\partial \bar{x}} + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho}\bar{u} - \frac{v}{k}\bar{u} \qquad (3)$$

where u and v are the velocity components in the x- and y- directions respectively, u is the velocity component in x-direction, t is the time taken, v is the kinematic viscosity, V is the suction/injection velocity, ρ is the field density, p is the pressure, σ is the electrical conductivity, B_0 is the applied magnetic field and k is the permeability coefficient.

For steady case, which is independent of time, then;

$$\frac{\partial \bar{u}}{\partial t} = 0 \qquad (4)$$

and

$$\frac{1}{\rho}\frac{\partial p}{\partial \bar{x}} = \mathrm{A} \qquad (5)$$

substituting (4) and (5) into (3), we have;

$$V\frac{\partial \bar{u}}{\partial \bar{y}} = -\mathrm{A} + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho}\bar{u} - \frac{v}{k}\bar{u} \qquad (6)$$

Introducing the following non-dimensional quantities.

$$x = \frac{\bar{x}}{h}, \qquad y = \frac{\bar{y}}{h}, \qquad \frac{\partial}{\partial \bar{y}} = \frac{\partial}{\partial y} \cdot \frac{\partial y}{\partial \bar{y}}$$

$$u = \frac{\bar{u}}{\mathrm{A}_1 \frac{h}{V}} \Longrightarrow \bar{u} = u\mathrm{A}_1\frac{h}{V}$$

$$\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial}{\partial y}.u\mathrm{A}_1\frac{h}{V}$$

$$\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{A_1}{V}\frac{\partial u}{\partial y} \tag{7}$$

and

$$\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} = \frac{A_1}{hV}\frac{\partial^2 u}{\partial y^2} \tag{8}$$

The velocity (u) profile of the reactive hydromagnetic fluid flow for the steady state is obtained as follows:

Then, substituting (7) and (8) into (6), we have;

$$V.\frac{A_1}{V}\frac{\partial u}{\partial y} = -A + v\frac{A_1}{hV}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 A_1 h}{\rho V}u - \frac{vA_1 h}{kV}u \tag{9}$$

$$A_1\frac{\partial u}{\partial y} = -A + \frac{vA_1}{hV}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 A_1 h}{\rho V}u - \frac{vA_1 h}{kV}u \tag{10}$$

Multiplying each term by $\frac{hV}{vA_1}$,

and $A_1 = -A,$ we obtain;

$$\frac{hV}{v}\frac{\partial y}{\partial u} = \frac{hV}{v} + \frac{\partial^2 u}{\partial y^2} - \frac{\partial B_0^2 h^2}{\rho v}u - \frac{h^2}{k}u \tag{11}$$

Also, taking the dimensionless form of the steady state, where

$$R = \frac{hV}{v},\ M^2 = \frac{\sigma B_0^2 h}{\rho V},\ Da = \frac{k}{h^2},\ u = \frac{V}{v} \tag{12}$$

Therefore; using the above dimensionless quantities, equation (11) becomes;

$$R\frac{\partial u}{\partial y} = R + \frac{\partial^2 u}{\partial y^2} - M^2 hu - \frac{1}{Da}u \tag{13}$$

This implies that;

$$\frac{\partial^2 u}{\partial y^2} - R\frac{\partial u}{\partial y} - (M^2 R + \frac{1}{Da})u = -R \tag{14}$$

and taking the boundary conditions to become

$u(0) = 0$ and $u(1) = 0$, we solve (14) accordingly and discuss in the next section.

Analytical Solution

This section gives the analytical solution of the equation (14) for the velocity profile of a steady state for the reactive hydromagnetic fluid flow.

Using the boundary condition (12), the velocity profile becomes

$$u(y) = C_1 e^{m_1 y} + C_2 e^{m^2 y} + \frac{R}{RM^2 + \frac{1}{Da}} \tag{3.1}$$

where,

$$m_1 = \frac{1}{2}[R \pm \sqrt{R^2 + 4(RM^2 + \frac{1}{Da})}] \tag{15}$$

$$m_2 = \frac{1}{2}[R \pm \sqrt{R^2 + 4(RM^2 + \frac{1}{Da})}] \tag{16}$$

$$C_1 = \frac{1}{e^{m_2} - e^{m_1}}[-u_{B_2} + u_{B_1} e^{m_2} - \frac{R}{RM^2 + \frac{1}{Da}}(e^{m_2} - 1)] \tag{17}$$

$$C_2 = \frac{1}{e^{m_2} - e^{m_1}}[u_{B_2} - u_{B_1} e^{m_2} - \frac{R}{RM^2 + \frac{1}{Da}}(1 - e^{m_1})] \tag{18}$$

and the slip velocities are given by:

$$u_{B_1} = \frac{D_2E_2 - D_4E_1}{D_1D_4 - D_3D_2}, \qquad u_{B_2} = \frac{D_3E_1 - D_1E_2}{D_1D_4 - D_3D_2}, \tag{19}$$

where;

$$D_1 = \frac{m_1e^{m_2} - m_2e^{m_1}}{e^{m_2} - e^{m_1}}, \tag{20}$$

$$D_2 = \frac{m_2 - m_1}{e^{m_2} - e^{m_1}} \tag{21}$$

$$D_3 = (m_1 - m_2)e^{m_1 + m_2} \quad , \tag{22}$$

$$D_4 =, \frac{m_2e^{m_2} - m_1e^{m_1}}{e^{m_2} - e^{m_1}} + m\sigma \tag{23}$$

$$E_1 = R[\frac{1}{M^2R + \frac{1}{Da}}\{\frac{m_2(e^{m_1} - 1)}{e^{m_2} - e^{m_1}} - \frac{m_1(e^{m_2} - 1)}{e^{m_2} - e^{m_1}}\} + \frac{m}{\sigma}] \tag{24}$$

$$E_2 = R[\frac{1}{M^2R + \frac{1}{Da}}\{\frac{m_2e^{m_2}(e^{m_1} - 1)}{e^{m_2} - e^{m_1}} - \frac{m_1e^{m_1}(e^{m_2} - 1)}{e^{m_2} - e^{m_1}}\} - \frac{m}{\sigma}] \tag{25}$$

Discussion of Results

This section deals with the effect of various parameters on the velocity and temperature profiles of a steady state of reactive hydromagnetic fluid flow through a porous medium between permeable beds. This is calculated analytically and presented graphically. Steady velocity profiles (Figures 2, 3, 4) has been considered for various σ, Da^{-1} and M for corresponding fixed α, R, Da^{-1} and *M.*

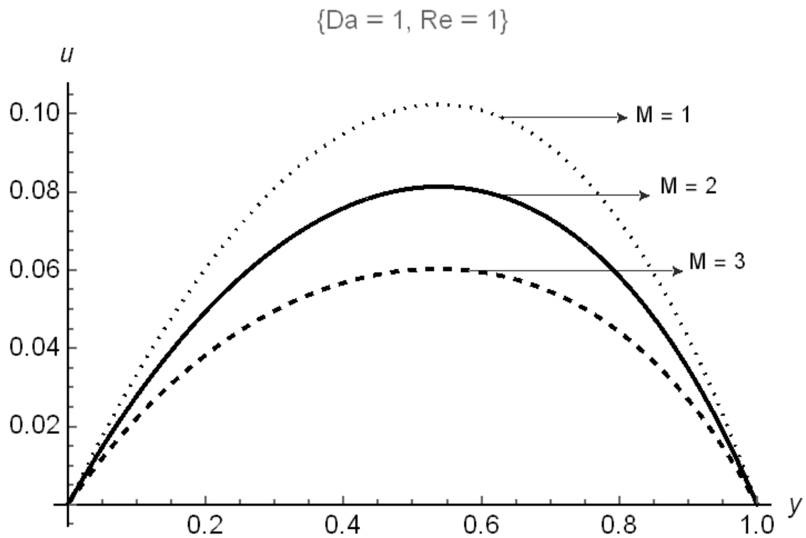

Figure 2: Velocity Profile with variations in magnetic term

The maximum flow is observed at the centreline of the fluid channel. Figure 2 displays the graph of momentum equation with variation in “Hartman M” number, it is clearly observed that as the *M* increases, the velocity of the flow reduces due to the magnetic influence retarding the fluid flow while maximum flow is observed.

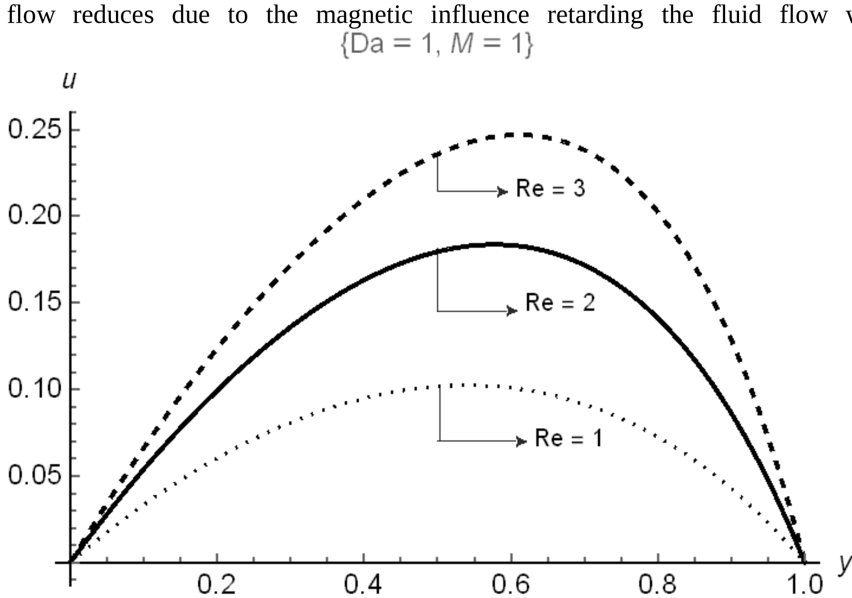

Figure 3: Velocity Profile with variations in R

Figure 3 shows that maximum velocity is recorded at the centerline of the flow channel. Also, it is observed that an increase in R brings about an increase in fluid velocity.

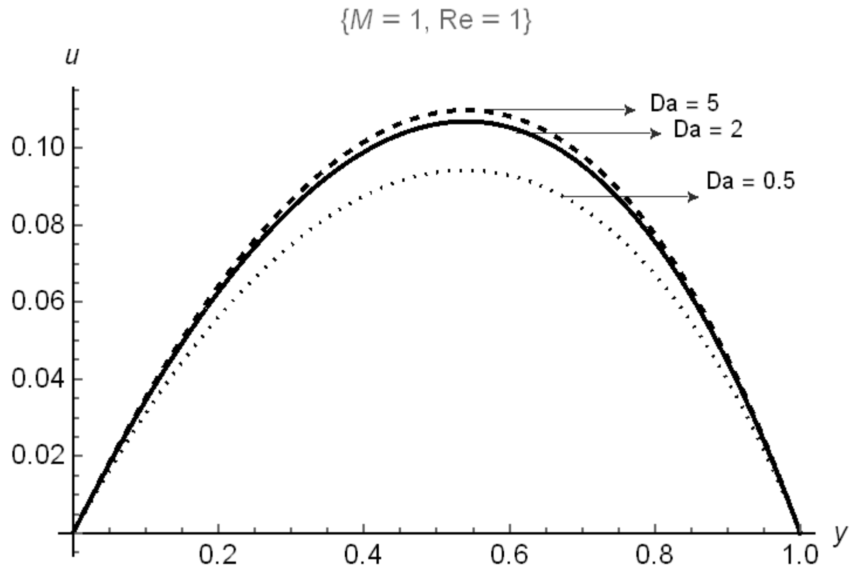

Figure 4: Velocity Profile with variations in Darcy term

Figure 4 also shows that as Darcy number increases, the velocity also increases. The reason is due to the inverse nature of Darcy permeability influence.

Conclusions and Recommendations

The analysis of a reactive hydromagnetic fluid flow through a porous medium between permeable pipe was carried out. A direct method was used to obtain the analytical solution. The results are discussed and the following were observed that the magnetic strength in the fluid channel reduces the velocity of the fluid flow due to the fact that the magnetic influence is retarding the fluid flow while the maximum flow is observed at the centerline. Also, an increase in the Reynolds number brings about an increase in the fluid velocity and the inverse Darcy permeability coefficient increases as the velocity profile of fluid flow increases. Finally, our results are in good agreement with the existing results in the literature.

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