Mathematical Modelling of Blood Flow Through a Stenosed Human Carotid Artery

M.A. Usman and S.A. Onitilo
Department of Mathematical Sciences, Olabisi Onabanjo University, Ago-Iwoye, Nigeria.
dekunle12@yahoo.com

Abstract

This paper investigates the mathematical modelling of blood flow through a stenosed human carotid artery. Constraints of blood flow in human carotid arteries are known as stenosis which may ends up into hypertension/stoke and heart failure. Also, the deposits of cholesterol on the arterial wall and proliferation of connective tissue are responsible for abnormal growth in the lumen of an artery. This work investigates the mathematical modelling of the effect of multistenosis in relation to shear stress, pressure gradient and viscosity of constricted human carotid arterial blood flow and its aftermath on the heart and general human health. The governing equations of blood flow in the human carotid artery were derived. The equations that were involved are the variables of interest such as number (n) of stenosis, percentage of Hematocrit (H) of red blood cels in the blood and the length (z) of the artery. Guided by medical data collected in the constraint of blood flow in a stenosed human carotid arteries, the governing equations were used to check the effects of pressure gradients, wall shear stress, velocity and volumetric flow rate of blood in human carotid arteries with the help of the boundary conditions. Observations revealed that; as Hematocrit and viscosity increase, the arterial wall shear stress decreases, an indication of increase in human heart pressure. However, increase in Hematocrit (H) and the length of the artery (z = 1) is inversely proportional to the arterial wall shear stress, this signifies the damage of veins around the arteries. The blood pressure gradient increases, which is directly proportional to the length of the artery (z) and Hematocrit, this suggest clotting of blood in human heart which can lead to death.

Keywords: stenosis; carotid artery; hematocrit of red blood cells; blood viscosity; wall shear stress; blood; blood vessels.

Introduction

Healthcare problems have apparently been of great concern to people these days. For over centuries, cardiovascular diseases have been identified as one of the major illness from which numerous people suffer. Stenosis or arteriosclerosis is the abnormal and unnatural growth in the arterial wall thickness that develops at various locations of the cardiovascular system under diseased conditions. This can be caused by unhealthy living conditions such as exposure to tobacco smoke, lack of physical activity and improper dietary habits. Carotid artery stenosis is a narrowing of the large arteries on either side of the week that carry blood to the head, face and brain. The narrowing is usually the result of a build-up of plague within the arteries, a condition called atherosclerosis. Stenosis can worsen over time to completely block the artery which may lead to stroke.

Literature illustrates few investigations on study of hemodynamic in carotid artery, Drikakis et al. (2011) considered simplest form of blood flow during reconstructive surgery. Blood was considered as non-Newtonian fluid in their study. Stenosed geometry was constructed as a function of upstream length in another study by Neofyton and Drikakis (2003). Stenosis was made independent of position in circulatory system. Morbiducci et al (2011) had calculated blood rheology marker for blood flow in models of carotid bifurcation. Newtonian rheology was hold true for bulk flow metrics and found to influence wall shear stress at different levels of geometry. Flow disturbance in carotid artery containing stenosis was also evaluated by Tan et al (2008). Gallo et al (2012) had formulated helical-flow of blood in region of carotid bifurcation. This phenomenon was used as surrogate marker for prediction of disturbed shear. Helicity-based bulk flow description was used to calculate the regions potential for exposure to disturbed shear. Liu et al (2000) described a numerical simulation of viscous flow in collapsible tubes with stenosis. Steinman et al (2000) described flow patterns at the stenosed carotid bifurcation, and has also shown the effect of concentric versus eccentric stenosis. Numerical analysis of flow through a severely stenotic carotid artery bifurcation was established by Stroud, et al (2002). Lee, et al (2002) gave a model from the behaviour of the flow and the wall in a mildly stenosed tube.

Srivastava, (2002) observed that the resistance by flow decreases with increasing shape parameter but increases with hematocrit (red cell concentration). Sing and sing (2012) studied the blood flow through radically non-symmetric stenosed artery and observed that resistance to flow increase as stenosis height or yield stress increases and decreases as stenosis shape increases and they quite hematocrit effect in blood flow modelling. Sriyab (2014) investigated the resistance and skin friction by K-L method in stenosis artery in presence of non-Newtonian blood with the Casson model which showed that skin function increases with the increasing of stenosis length bud did not investigate the effect of hematocrit level.

From the above discussion, it is clear that hematocrit plays important role in the blood system. So, the objective of this study is to investigate the mathematical modelling of blood flow through a stenosed human carotid artery with the effects of hematocrit in the blood flow system.

Mathematical Modelling

In developing the mathematical model for blood flow, the following assumptions are taken into consideration; the flow is an incompressible, the fluid is non-Newtonian, the density is constant, the viscosity of blood varies radically and the stenosis is mild.

The schematic diagram of blood flow in the artery is shown in figure 1. The problem has been studied cylindrical coordinate system (r, θ, z) where z axis is taken along the axis of the artery while r and θ are along the radial and the circumferential directions respectively.

The geometry of the stenosis in one-dimensional form which develops symmetrical about the artery axis but non-symmetric with respect to radial coordinate is given as

$$\frac{R(z)}{R_0} = 1 - b[(S_L)^{k-1}\{z - (d_1 + d_2)\} - \{z - (d_1 + d_2)\}^k]$$

$$(d_1 + d_2) \leq z \leq (d_1 + d_2) + S_L \tag{1}$$

where $b = \frac{\delta}{R_0(S_L)^k}$

Where d_1, d_2 are distance of the stenosis region, z is the length of the artery, δ is the maximum height of the stenosis, R_0 is the radius of the vessel without stenosis and $R(z)$ is radius of the vessel with stenosis.

Consider the one-dimensional equation for the steady and axially symmetric flow of blood through an artery provided with a mild stenosis under the above-mentioned assumption is:

$$\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}[\tau r] = 0 \tag{2}$$

Where p is the fluid pressure, τ is the shear stress, $[\tau r]$ are the cylindrical polar co-ordinates with z measured along the stenosis axis and r measured normal to the axis of the stenosis.

According to Lih (1969), the Einstein coefficient of viscosity of blood was given as:

$$\mu^*(r) = \mu_0[1 + ch^*(r)] \tag{3}$$

where μ_0 is the coefficient of viscosity of plasma, $c = 2.5$ and $h^*(r)$ is the hematocrit defined as:

$$h^*(r) = H\left[1 - \left(\frac{r}{R_0}\right)^n\right] \tag{4}$$

The boundary conditions are:

$$\begin{aligned} u &= 0, \quad at \quad r = R(z) \\ \frac{du}{dr} &= 0, \quad at \quad r = 0 \end{aligned} \tag{5}$$

Suppose $x = \frac{r}{R_0}$ (6)

which is the radial coordinate, where R_0 is the inner radius of the vessel and x is the arterial wall viscosity. Then, using equations (3), (4) and (6) in the transformation of equation (2) with the following procedures

$$\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}[\tau r] = 0$$

Substituting equations (3), into (4) gives

$$\mu^*(r) = \mu_0\left[1 + cH\left(1 - \left(\frac{r}{R_0}\right)^n\right)\right]$$

$$\mu^*(r) = \mu_0\left[1 + cH - cH\left(\frac{r}{R_0}\right)^n\right] \tag{7}$$

Thus equation (7) is therefore expressed as

$$\mu^*(r) = \mu_0[m_1 - m_2 x^n] \tag{8}$$

where $m_1 = 1 + m_2, \qquad m_2 = cH$ (9)

But $\tau = -\mu^*\frac{du}{dr}$ (10)

Substituting equation (8), into (10) gives

$$\tau = -\mu_0[m_1 - m_2 x^n]\frac{du}{dr} \tag{11}$$

Using equation (6), where $r = xR_0$ into equation (11)

From equation (6), i.e. $x = \frac{r}{R_0}$

$$r = xR_0, \text{ therefore } \frac{du}{dr} = \frac{1}{R_0}\frac{du}{dx}$$

$$\tau = -\mu_0[m_1 - m_2 x^n]\frac{1}{R_0}\frac{du}{dx} \tag{12}$$

Substituting equations (6) and (12) into equation (2) yields

$$\frac{\partial p}{\partial z} = \frac{\mu_0}{R_0^2}\frac{1}{x}\frac{d}{dx}[x(m_1 - m_2 x^n)]\frac{du}{dx} \tag{13}$$

Multiplying equation (13) by $\frac{R_0^2}{\mu_0}$, gives:

$$\frac{R_0^2}{\mu_0}\frac{\partial p}{\partial z} = \frac{1}{x}\frac{d}{dx}\left[x(m_1 - m_2 x^n)\frac{du}{dx}\right] \tag{14}$$

Further simplification gives

$$\frac{R_0^2}{\mu_0}\frac{\partial p}{\partial z} = \frac{1}{x}\frac{d}{dx}\left[x(m_1 - m_2 x^n)\frac{du}{dx}\right] \tag{15}$$

The boundary condition in equation (5) becomes

$$u = 0, \quad at \quad x = \frac{R(z)}{R_0}$$
$$\frac{du}{dx} = 0, \quad at \quad x = 0 \tag{16}$$

Equation (15) and the boundary conditions in equations (16) are applicable only when $n \geq 2$.

Method of Solution

Equation (15) is solved with the help of boundary conditions (16), which gives:

$$\frac{d}{dx}\left[x(m_1 - m_2 x^n)\frac{du}{dx}\right] = \frac{xR_0^2}{\mu}\frac{dp}{dz} \tag{17}$$

Integration of equation (17) gives

$$x(m_1 - m_2 x^n)\frac{du}{dx} = \frac{x^2 R_0^2}{2\mu}\frac{dp}{dz} + C \tag{18}$$

where C is the constant of integration

Noting that $\frac{du}{dx} = 0, \ at \ x = 0,$ then $c = 0.$ (19)

Therefore, equation (18) becomes

$$x(m_1 - m_2 x^n)\frac{du}{dx} = \frac{x^2 R_0^2}{2\mu}\frac{dp}{dz} \tag{20}$$

Solving equation (20) with the boundary condition in equation (16) analytically we have

$$u(x) = \frac{m_1}{2m_2}\left[\left(\frac{R(z)}{R_0}\right)^{-2} - \frac{1}{x^2}\right] + \frac{R_0^2}{2\mu_0 m_2}\frac{dp}{dz}\left[\frac{1}{x} - \left(\frac{R(z)}{R_0}\right)^{-1}\right] \tag{21}$$

which is the velocity of the blood.

The volumetric flow rate Q is given by:

$$Q = 2\pi R_0 \int_0^{R/R_0} x\, u(x) dx \tag{22}$$

Hence, substituting equation (21) into equation (22) yields

$$Q = 2\pi R_0 \int_0^{R/R_0} x \left\{\frac{m_1}{2m_2}\left[\left(\frac{R(z)}{R_0}\right)^{-2} - \frac{1}{x^2}\right] + \frac{R_0^2}{2\mu_0 m_2}\frac{dp}{dz}\left[\frac{1}{x} - \left(\frac{R(z)}{R_0}\right)^{-1}\right] dx\right\} \tag{23}$$

$$Q = 2\pi R_0 \left[\int_0^{R/R_0} x \frac{m_1}{2m_2}\left\{\frac{1}{\left(\frac{R(z)}{R_0}\right)^2} - \frac{1}{x^2}\right\} dx\right] + \frac{\pi R_0^3}{\mu_0 m_2}\frac{dp}{dz}\left[\int_0^{R/R_0}\left\{1 - \frac{x}{\left(\frac{R(z)}{R_0}\right)}\right\} dx\right]$$

$$Q = \frac{\pi R_0 m_1}{m_2}\left[\frac{1}{\left(\frac{R(z)}{R_0}\right)^2}\left[\frac{x^2}{2}\right]_0^{R/R_0}\right] - \frac{\pi R_0 m_1}{m_2}[log x]_0^{R/R_0} + \frac{\pi R_0^3}{\mu_0 m_2}\cdot\frac{dp}{dz}\left\{[x]_0^{R/R_0} - \frac{1}{\frac{R(z)}{R_0}}\left[\frac{x^2}{2}\right]_0^{R/R_0}\right\} \tag{24}$$

$$Q = \frac{\pi R_0 m_1}{m_2}\left\{\frac{1}{\left(\frac{R(z)}{R_0}\right)^2}\cdot\left[\frac{\left({}^{R}/{}_{R_0}\right)^2}{2}\right]\right\} - \frac{\pi R_0 m_1}{m_2}\log\left(\frac{R(z)}{R_0}\right) + \frac{\pi R_0^3}{\mu_0 m_2}\cdot\frac{dp}{dz}\left(\frac{R(z)}{R_0}\right) - \frac{\pi R_0^3}{\mu_0 m_2}\cdot\frac{dp}{dz}\cdot\frac{1}{\left(\frac{R(z)}{R_0}\right)}\frac{\left(\frac{R(z)}{R_0}\right)^2}{2} \tag{25}$$

$$Q = \frac{\pi R_0 m_1}{2m_2} - \frac{\pi R_0 m_1}{m_2} log\left(\frac{R(z)}{R_0}\right) + \frac{\pi R_0^2}{\mu_0 m_2} R(z)\frac{dp}{dz} \tag{26}$$

If Q_0 is the flow rate of plasma fluid in the unconstricted tube, then

$$Q_0 = -\frac{\pi R_0^4}{8\mu}\left(\frac{dp}{dz}\right)_0 \tag{27}$$

Let $\frac{Q}{Q_0} = 1$; then $Q = Q_0$

that is $$\frac{\pi R_0 m_1}{2m_2} - \frac{\pi R_0 m_1}{m_2}\log\left(\frac{R(z)}{R_0}\right) + \frac{\pi R_0^2}{\mu_0 m_2} R(z)\frac{dp}{dz} = -\frac{\pi R_0^4}{8\mu}\left(\frac{dp}{dz}\right)_0 \tag{28}$$

$$\frac{\pi R_0^2}{\mu_0 m_2} R(z)\frac{dp}{dz} = \frac{\pi R_0 m_1}{m_2} log\left(\frac{R(z)}{R_0}\right) - \frac{\pi R_0 m_1}{2m_2} - \frac{\pi R_0^4}{8\mu}\left(\frac{dp}{dz}\right)_0 \tag{29}$$

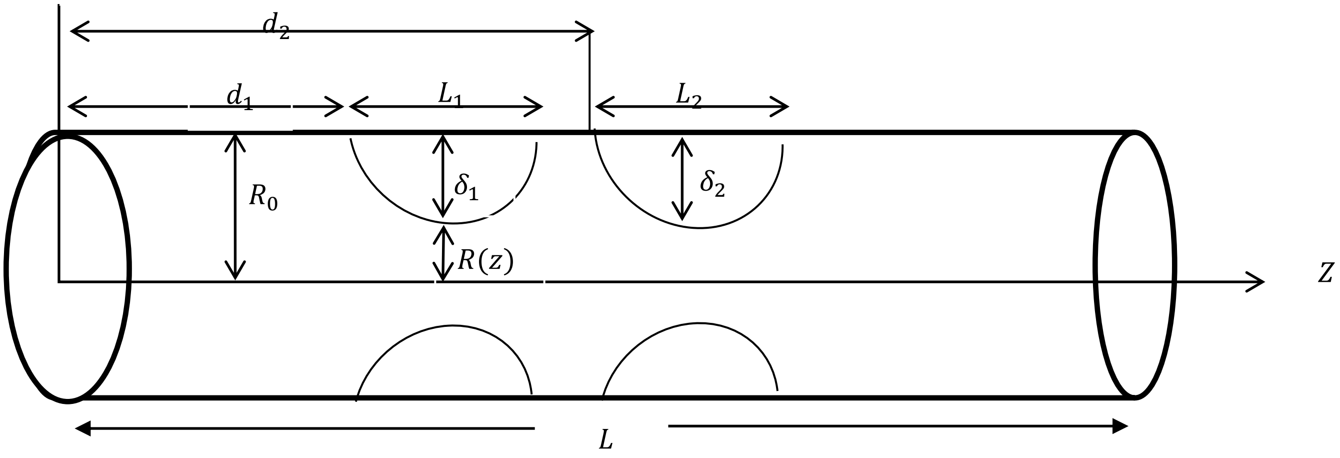

Figure 1: Schematic Diagram of Blood Vessels with Stenosis

For $n = 2$, putting $S_L = 1,\ k = 2,\ d_1 = {}^1/_2,\ d_2 = {}^1/_4,\ b = {}^1/_2$ into equation (1)

We have

$$\frac{R(z)}{R_0} = 1 - \frac{1}{2}\left[1\{z - ({}^1/_2 + {}^1/_4)\} - \{z - ({}^1/_2 + {}^1/_4)\}^2\right] \tag{30}$$

$$\frac{R(z)}{R_0} = 1 - \frac{1}{2}\left[(z - {}^3/_4) - (z - {}^3/_4)^2\right]$$

$$\frac{R(z)}{R_0} = 1 - \frac{1}{2}\left[\frac{40z - 16z^2 - 21}{16}\right]$$

$$\frac{R(z)}{R_0} = \frac{16z^2 - 40z + 53}{32} \tag{31}$$

$$\frac{R(z)}{R_0} = \frac{\alpha}{32}, \tag{32}$$

Where $\alpha = 16z^2 - 40z + 53$

Taking the logarithm of both sides, we have

$$log\left(\frac{R(z)}{R_0}\right) = log\left(\frac{\alpha}{32}\right) \tag{33}$$

Putting equations (32) and (33) into equation (34), gives

$$\frac{\pi R_0^3}{\mu_0 m_2} \cdot \frac{\alpha}{32} \cdot \frac{dp}{dz} = \frac{\pi R_0 m_1}{m_2} log\left(\frac{\alpha}{32}\right) - \frac{\pi R_0 m_1}{2m_2} - \frac{\pi R_0^4}{8\mu}\left(\frac{dp}{dz}\right)_0 \tag{34}$$

Dividing equation (34) through by π, yields

$$\frac{\alpha}{32} \cdot \frac{R_0^3}{\mu_0 m_2} \cdot \frac{dp}{dz} = \frac{R_0 m_1}{m_2} log\left(\frac{\alpha}{32}\right) - \frac{R_0 m_1}{2m_2} - \frac{R_0^4}{8\mu}\left(\frac{dp}{dz}\right)_0 \tag{35}$$

Dividing equation (35) through by $\left(\frac{dp}{dz}\right)_0$,

that is $\left(\frac{dp}{dz}\right)_0 = -\frac{8\mu Q_0}{\pi R_0^4}$ (36)

$$\frac{\alpha}{32} \cdot \frac{R_0^3}{\mu_0 m_2} \cdot \frac{dp/dz}{\left(dp/dz\right)_0} = \frac{R_0 m_1}{m_2} \cdot \frac{1}{\left(dp/dz\right)_0} \cdot log\left(\frac{\alpha}{32}\right) - \frac{1}{\left(dp/dz\right)_0}\frac{R_0 m_1}{2m_2} - \frac{R_0^4}{8\mu} \tag{37}$$

Simplifying (37) further gives

$$\frac{dp/dz}{\left(dp/dz\right)_0} = \frac{32 m_1 \mu_0}{\alpha R_0^2} \cdot \frac{1}{\left(dp/dz\right)_0} log\left(\frac{\alpha}{32}\right) - \frac{16 m_1 \mu_0}{\alpha R_0^2}\frac{1}{\left(dp/dz\right)_0} - \frac{4 m_2 \mu_0 R_0}{\alpha \mu} \tag{38}$$

Substituting $\frac{1}{\left(dp/dz\right)_0} = -\frac{\pi R_0^4}{8\mu Q_0}$ into equation (38) to obtain

$$\frac{dp/dz}{\left(dp/dz\right)_0} = -\frac{32 m_1 \mu_0}{\alpha R_0^2} \cdot \frac{\pi R_0^4}{8\mu Q_0} log\left(\frac{\alpha}{32}\right) + \frac{2\pi R_0^2 m_1 \mu_0}{\alpha \mu Q_0} - \frac{4 m_2 \mu_0 R_0}{\alpha \mu} \tag{39}$$

Further simplification of equation (39) gives

$$\frac{dp/dz}{\left(\frac{dp}{dz}\right)_0} = \frac{4 m_2 R_0}{\alpha Q_0}\left[-\frac{\pi R_0 m_1 \mu_0}{\mu m_2} log\left(\frac{\alpha}{32}\right) + \frac{\pi R_0 m_1 \mu_0}{2\mu m_2} - \frac{\mu_0}{\mu} Q_0\right] \tag{40}$$

Putting $\mu = \mu_0$ and $Q = Q_0$ into equation (40) yields

$$\frac{{dp}/{dz}}{\left(\frac{dp}{dz}\right)_0} = \frac{4R_0m_2}{\alpha Q_0}\left[\frac{\pi R_0 m_1}{2m_2} - Q - \frac{\pi R_0 m_1}{m_2} log\left(\frac{\alpha}{32}\right)\right] \tag{41}$$

Therefore, equation (41) is the pressure gradient of the blood flow.

The wall shear stress of artery is defined by

$$\tau_w = \mu\left(\frac{du}{dr}\right)_{r=R(z)} \tag{42}$$

If $$u = \frac{m_1}{2m_2}\left[\left(\frac{R(z)}{R_0}\right)^{-2} - \frac{1}{x^2}\right] + \frac{R_0^2}{2\mu_0 m_2}\cdot\frac{dp}{dz}\left[\frac{1}{x} - \left(\frac{R(z)}{R_0}\right)^{-1}\right] \tag{43}$$

Since $x = \frac{r}{R_0}$,

Therefore $$\frac{x}{r} = \frac{1}{R_0} \tag{44}$$

Equation (43) can be re-written as

$$u = \frac{m_1}{2m_2}\left[\left(\frac{(R_0)}{R_{(z)}}\right)^{2} - \frac{1}{x^2}\right] + \frac{R_0^2}{2\mu_0 M_2}\frac{dp}{dz}\left[\frac{1}{x} - \left(\frac{R_0}{R(z)}\right)\right] \tag{45}$$

Substituting equation (44) into equation (45) gives

$$u = \frac{m_1}{2m_2}\left[\frac{r^2}{x^2}\left(\frac{1}{R(z)}\right)^{2} - \frac{1}{(x)^2}\right] + \frac{R_0^2}{2\mu_0 M_2}\frac{dp}{dz}\left[\frac{1}{x} - \frac{r}{x}\left(\frac{1}{R(z)}\right)\right]$$

Hence,

$$u = \frac{m_1}{2m_2}\left[\frac{r^2}{x^2(R(z))^2} - \frac{1}{x^2}\right] + \frac{R_0^2}{2\mu_0 m_2}\frac{dp}{dz}\left[\frac{1}{x} - \frac{r}{xR(z)}\right] \tag{46}$$

Differentiating equations (46), to obtain

$$\frac{du}{dr} = \frac{m_1 r}{m_2 x^2(R(z))^2} - \frac{R_0^2}{2\mu_0 m_2}\frac{dp}{dz}\left(\frac{1}{xR(z)}\right) \tag{47}$$

Substituting equation (47) into equation (42), gives

$$\tau_w = \mu\left[\frac{m_1 r}{m_2 x^2(R(z))^2} - \frac{R_0^2}{2\mu_0 m_2}\frac{dp}{dz}\left(\frac{1}{xR(z)}\right)\right] \tag{48}$$

Since $r = R(z)$, then

$$\tau_w = \mu\left[\frac{m_1}{m_2 x^2 r} - \frac{R_0^2}{2\mu_0 m_2}\frac{dp}{dz}\left(\frac{1}{xr}\right)\right] \tag{49}$$

From equation (3), $\mu = \mu_0(m_1 - m_2 x^n)$

Putting $n = 2$ into equation (8)

$$\mu = \mu_0(m_1 - m_2 x^2) \tag{50}$$

Substituting equation (50) into equation (48) yields

$$\tau_w = [\mu_0(m_1 - m_2x^2)]\left[\frac{m_1}{m_2x^2r} - \frac{R_0^2}{2\mu_0 m_2}\cdot\frac{1}{xr}\left(\frac{dp}{dz}\right)\right] \tag{51}$$

Hence,

$$\tau_w = \frac{\mu_0}{x^3}(m_1 - m_2x^2)\left[\frac{m_1x}{m_2r} - \frac{x^2R_0^2}{2r\mu_0 m_2}\frac{dp}{dz}\right] \tag{52}$$

Noting that $\frac{1}{R_0} = \frac{x}{r}$ from equation (44), then, equation (52) becomes

$$\tau_w = \frac{\mu_0}{x^3}(m_1 - m_2x^2)\left[\frac{m_1}{m_2R_0} - \frac{R_0x}{2\mu_0 m_2}\frac{dp}{dz}\right] \tag{53}$$

If $\boldsymbol{\tau_N}$ is the shear stress of plasma fluid at the normal artery wall, then it is defined as

$$\tau_N = \left(-\frac{R_0}{2}\right)\left({}^{dp}/_{dz}\right)_0 \tag{54}$$

Therefore, the shear stress of blood flow is

$$\tau = \frac{\tau_w}{\tau_N} \tag{55}$$

Substituting equations (53) and (54) into equation (55) to obtain

$$\tau = \frac{2\mu_0}{x^3R_0}(m_2x^2 - m_1)\left[\frac{m_1}{m_2R_0\left({}^{dp}/_{dz}\right)_0} - \frac{R_0^2x}{2\mu_0m_2}\frac{{}^{dp}/_{dz}}{\left({}^{dp}/_{dz}\right)_0}\right] \tag{56}$$

But $r = R(z)$, and $x = \frac{r}{R_0} = \frac{R(z)}{R_0} = \frac{\alpha}{32}$ (57)

Substituting equation (57) into equation (56) gives

$$\tau = \frac{2\mu_0}{R_0^2}\left(\frac{m_2}{{}^{R(z)}/_{R_0}} - \frac{m_1}{\left(\frac{R(z)}{R_0}\right)^3}\right)\left[\frac{m_1}{m_2\left({}^{dp}/_{dz}\right)_0} - \frac{{}^{R(z)}/_{R_0}\cdot R_0}{2\mu_0m_2}\frac{{}^{dp}/_{dz}}{\left({}^{dp}/_{dz}\right)_0}\right]$$

$$= \frac{2\mu_0}{R_0^2}\left[\frac{32m_2}{\alpha} - \left(\frac{32}{\alpha}\right)^3 m_1\right]\left[\frac{m_1}{m_2R_0\left({}^{dp}/_{dz}\right)_0} - \frac{\alpha}{32}\cdot\frac{R_0}{2\mu_0m_2}\frac{{}^{dp}/_{dz}}{\left({}^{dp}/_{dz}\right)_0}\right] \tag{58}$$

Equation (58) is further simplified to obtain

$$\tau = \frac{2\mu_0}{R_0^2}\left[\frac{32m_2}{\alpha} - \left(\frac{32}{\alpha}\right)^3 m_1\right]\left[\frac{m_1}{m_2R_0\left({}^{dp}/_{dz}\right)_0} - \frac{\alpha}{64}\cdot\frac{R_0}{\mu_0m_2}\frac{{}^{dp}/_{dz}}{\left({}^{dp}/_{dz}\right)_0}\right] \tag{59}$$

Results and Discussion

This chapter presents the results and discussion of this research work, the plots are shown for number of stenosis $(n = 2, 3, 4 \text{ and } 5)$. To be realistic, we have chosen physically meaningful value for $c = 2.5$, $S_L = 1$, $K = 2$, $d_1 = \frac{1}{2}$, $d_2 = \frac{1}{4}$, $b = \frac{1}{2}$, $z = 1$, and $n = 2$.

All graphs were drawn using MATLAB 2013.

The variation of wall shear stress and pressure gradient along the length of artery and viscosity for different values of Hematocrit (H) of red blood cell.

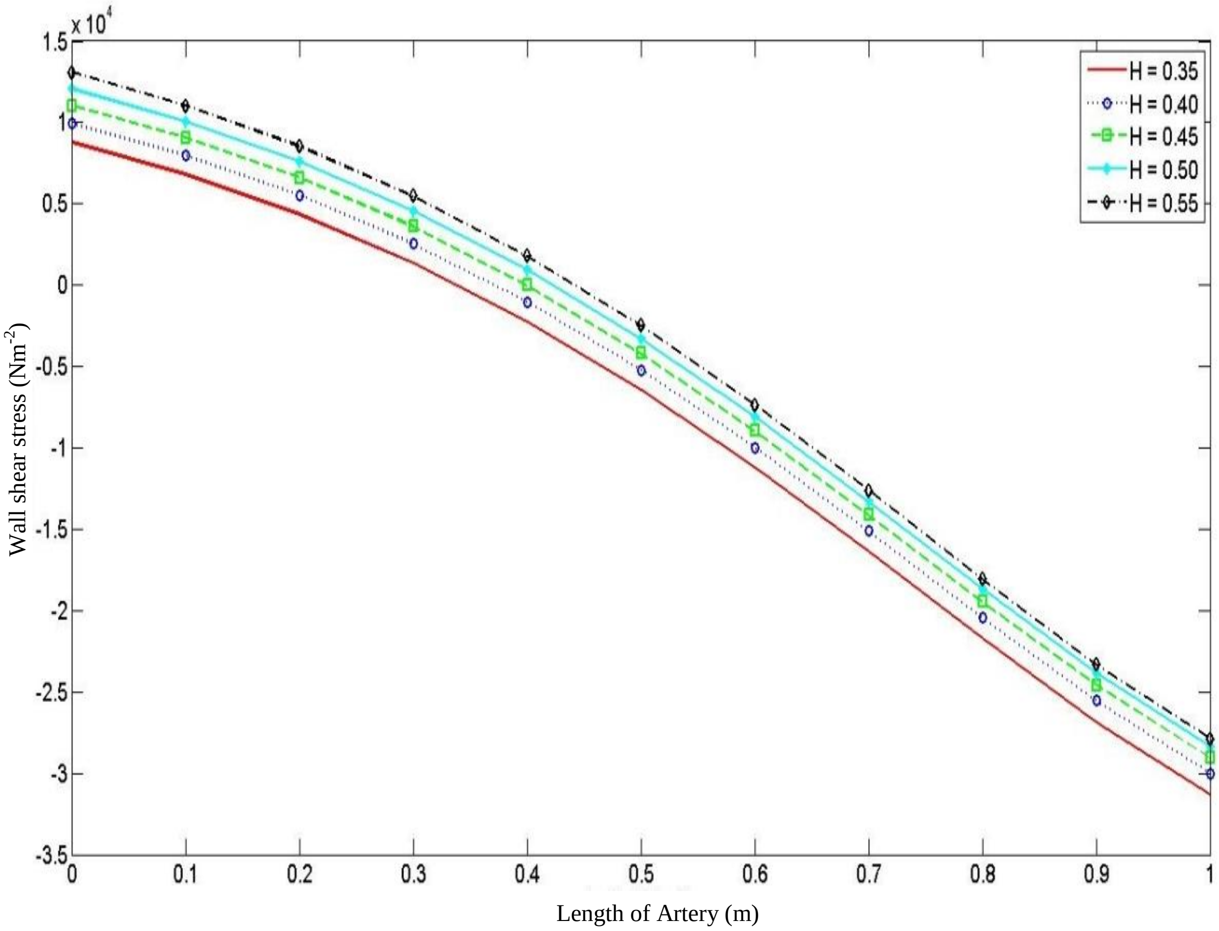

Figure 2: Variation of wall shear stress along length of artery (Z) for $\mu = 2$.

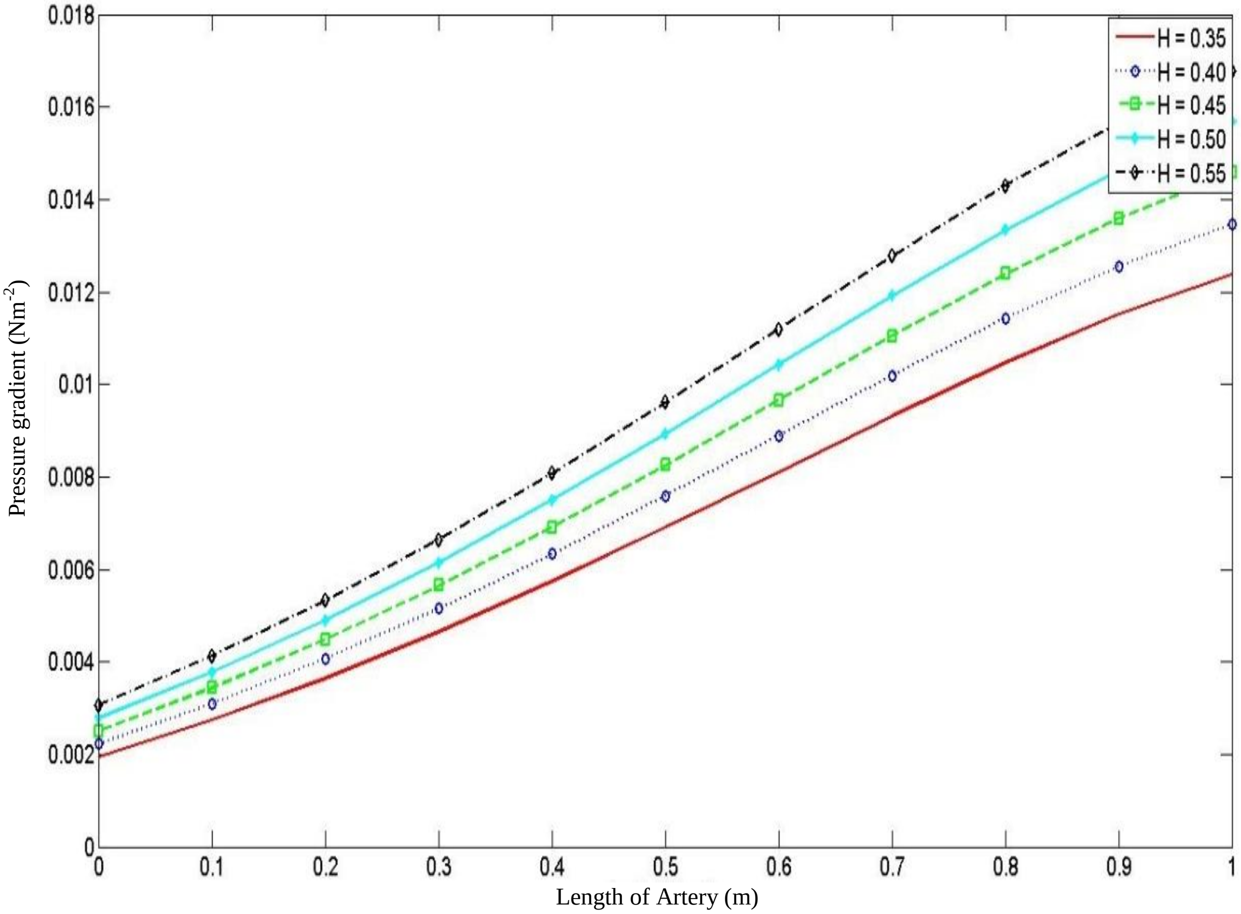

Figure 3: Variation of pressure gradient along length of artery (Z) for μ = 2.

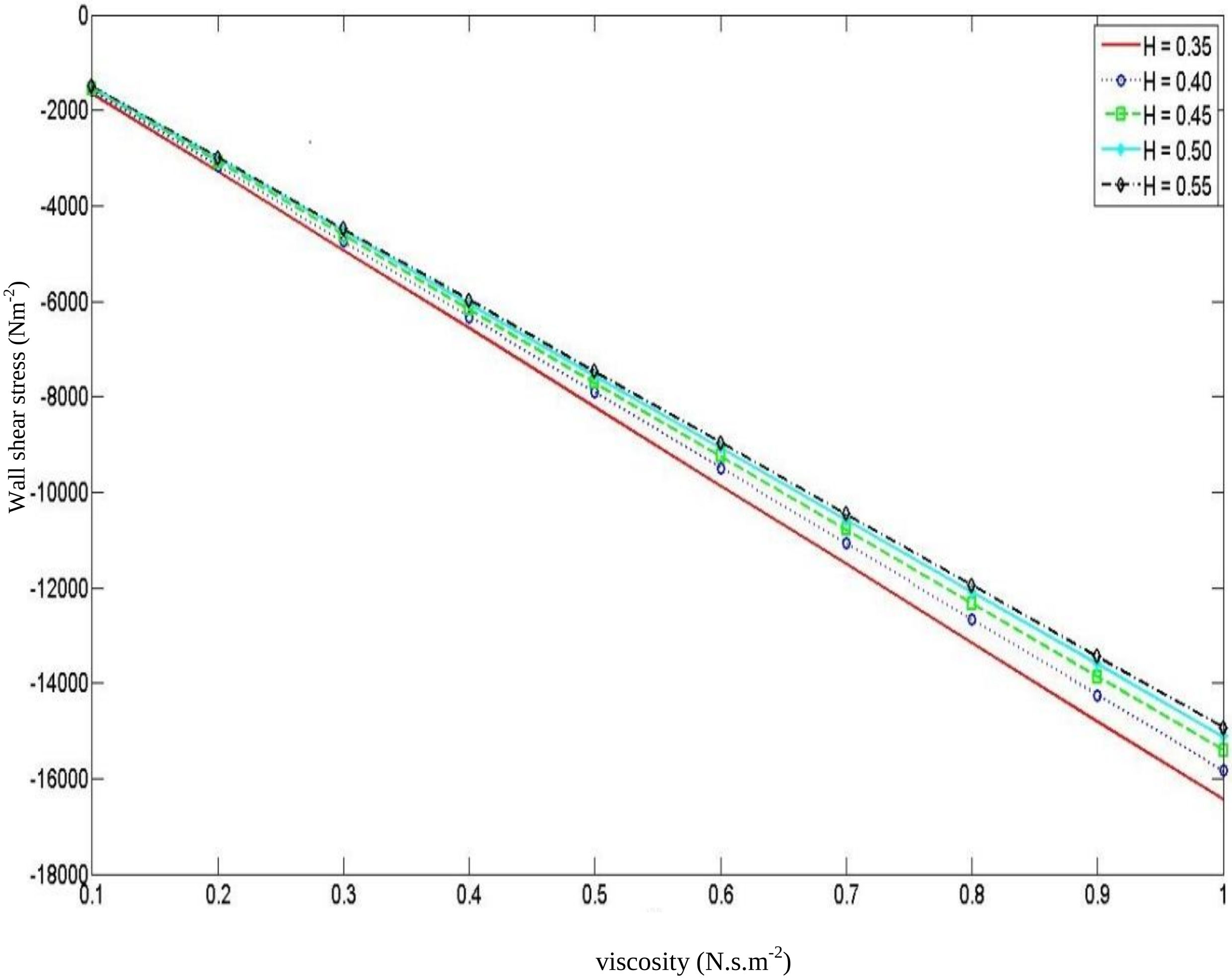

Figure 4: Variation of pressure gradient along viscosity (μ) for μ= 2

Discussion of Results

The variation of wall shear stress and pressure gradient along the length of human artery and viscosity of blood $(n = 2)$ for different value of Hematocrit (H) of red blood cell.

Figure 2 indicates the variation of wall shear stress along the length of artery for different value of Hematocrit (H) of red blood cell. It is observed that at a particular value of (Z=1) the rise of wall shear stress decreases with the increase in the value of Hematocrit (H) of red blood cell and the length of human artery. In figure 3 that shows the variation of pressure gradient along the length of human artery for different Hematocrit (H) of red blood cell and viscosity of blood. It is clear that the pressure gradient increases. This occur because of the increase in the viscosity of the blood and presence of stenosis which obstruct the blood flow in the artery and increase the force exerted on the wall of the tiny or flexible artery, which is contrary to Vipin and Praveen (2013) result that tested the hypothesis that the model for single stenosis is also valid for multiple stenosis arranged in series predicted for measurements of individual stenosis components.

Figure 4 indicate the variation of wall shear stress along the viscosity of blood for different value of Hematocrit (H) of red blood cell. It is clearly observed that for a value of blood viscosity the increases as the wall shear stress is significant for Hematocrit (H) of red blood cell. The wall shear stress increases as the blood viscosity and heamatocrit of red blood cell increases because of the presence of double stenosis in the human carotid artery which disturb the normal blood flow.

Conclusions and Recommendations

This work focused on the effect of Hematocrit on flow parameters of blood such as resistance, stenosis in the artery and fluid characteristics, taking blood as non-Newtonian fluid. It was found that resistance increased with the increasing of hematocrit level and stenosis height and number. It also showed that with increase in Hematocrit (H) of red blood cells, the pressure gradient, shear stress and viscosity of the blood increases. The results obtained indicates that increase in the height and number of stenosis in the artery could result in high blood pressure which can get to a critical region that is beyond control, which can cause bleeding or clotting of the blood which is very dangerous to human heart/brain as it can result into heart failure or stroke.

References

Srivastava, V.P. (2002). Particulate suspension blood flow through stenotic arteries: Effects of hematocrit and stenosis shape. *Indian J. Pure Appl. Math.*, 33(9), pp. 1353-1360.

Drikakis, D., Milionis, C., Pal, S.K., Patel S., and Shapiro, E. (2011). Assessment of the applicability of analytical models for blood flow prediction in reconstructive surgery. *International Journal for Numerical Methods in Biomedical Engineering,* 27(7), pp. 993-999.

Neofytou, P. and Drikakis, D. (2003). Effects of blood models on flows thorough a stenosis. *International Journal for Numerical Methods in Fluids,* 43(6-7), pp. 597-635.

Morbiducci, U., Gallo, D., Massal, D. (2011). On the importance of blood rheology for bulk flow in hemodynamic models of the carotid bifurcation. *Journal of Biomechanics,* 44(13), pp. 2427-2438.

Vipin, K.V. & Praveen, S. (2013). Effect of a multiple stenosis on blood flow through a tube. *International Journal of Medical, Health, Biomedical, Bioengineering and Pharmaceutical Engineering. 7(11), 753-756.*

Tan, F.P.P., Soloperto, G., Bashford S. (2008). Analysis of flow disturbance in a stenosed carotid artery bifurcation using two-equation transitional and turbulence models. *Journal of Biomechanical Engineering,* 130(6), p. 1-12.

Gallo, D., Steinman, D.A., Bijari, P.B. and Morbiducci, U. (2009). Helical flow in carotid bifurcation as surrogate marker of exposure to disturbed shear. *Journal of Biomechanics,* 45(14), pp. 2398-2404.

Steinman, D.A., Poepping, T.L., Tambasco, M., Rankin, R.N. and Holdsworth, D.W. (2000). Flow patterns at the stenosed carotid birfurcation: effect of concentric versus eccentric stenosis. *Annals of Biomedical Engineering,* 28(4), pp. 415-523.

Liu, B. and Tang, D. (1999). A Numerical Simulation of Viscous Flows in Collapsible Tubes with Stenoses. *Appl. Num. Math.,* 32(1), pp. 87-101.

Lee, K.W. and Xu, X.Y. (2002). Modeling of Flow and Wall Behaviour in a Mildly Stenosed Tube. *Med. Eng. Phys.,* 24(9), 575-586.

Stroud, J., Berger, S. and Saloner, D. (2002). Numerical Analysis of Flow through a Severely Stenotic Carotid Artery Bifurcation. *Journal of Biomechanical Engineering,* 158(1-2), pp. 155-196.

Sriyab, S. (2014). Mathematical Analysis of Non-Newtonian Blood Flow in Stenosis Narrow Arteries. *Comput Math Methods Med.,* 2014, p. 479152.

APPENDIX

$R(z)$	Radius of the artery at stenosed portion
R_0	Radius of the artery
L	Length of the artery
δ	The maximum height of the stenosis
z	Axial coordinate
p	The fluid pressure
(r,z)	The cylindrical polar co-ordinate with z measured along the axis of the stenosis
H	Hematocrit (H) of red blood cell
μ	Viscosity of the fluid
u	Velocity of the fluid
Q	Volumetric flow rate
τ	Wall shear stress
x	Arterial wall viscosity