A Study of Oscillatory Magnetohydrodynamics Flow of Dusty Viscoelastic Fluid Through an Inclined Channel with OHMIC Heating

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Abstract
In this research, an investigation study of the oscillatory MHD Poisuelle flow of dusty viscoelastic fluid down an inclined channel has been carried out. The fluid is electrically conducting containing dusty particle which are solid, elastic spheres, identical and symmetrical in size, electrically non-conducting and distributed uniformly within the fluid motion. Continuum equations governing laminar flow of the dusty viscoelastic fluid due to an oscillatory pressure gradient in the presence of a transverse magnetic field were developed. With the assumptions of incompressibility and constant particulate volume fraction the resulting equations were solved analytically via perturbation techniques. Numerical evaluations of the analytically solutions were computed and shown graphically to illustrate the effects of different physical parameters on the solutions. It is clearly show that income is the values of magnetic parameter Ha have the tendency to show the movement of the suspension in the channel, causing a reduction in the flow rate of both phases and in the fluid wall shear stress.The more interesting was the cross-over of the velocity curve of the fluid and the particles due to the variation of the suction parameter S and the influence of the magnetic field in the suspension of such cross-over.

Keywords: Dusty Viscoelastic Fluids; Poisuelle Flow; Non-Newtonian Fluid;Skin Friction; Magnetic Prandtl Number;Dust Particle Concentration.

Introduction

The study of the flow of dusty viscoelastic fluids has important applications with fluidization, combustion, use of dust in gas cooling systems, centrifugal system of matter from fluid, petroleum industry and purification of crude oil, electrostatic precipitation, polymer technology and droplets sprays.

The hydrodynamic flow of dusty fluid, was studied by a number of authors. Kannan and Ramamurthy (2001) applied perturbation technique to obtain inner and outer solutions for the problem of induced dusty flow due to normal oscillation of the wavy wall and have shown an interested application of their result to mechanical engineering for the possibility of fluid transportation without external pressure.

Venkataraman and Kannan (2004) discussed the two-phase fluid flow on an infinite vertical plate with viscous incompressible and dissipative heat. They discussed the effects of fluid flow for various values of Grashof and Eckert numbers. Also, they observed that skin friction values increased in dusty fluid, heat transfer rate decreased for increase in mass concentration. For higher values of t, particles never allowed the reverse flow.

Ghosh and Ghosh (2005) studied to analyze fluid – particle interaction mechanism in various types of flows. An initial value investigation is made of the motion of an incompressible, viscous, conducting fluid with embedded small inert spherical particles bounded by an infinite rigid non-conducting plate. The unsteady flow is supposed to generate from rest in the fluid-particle system due to velocity tooth pulses being imposed on the plate in presence of a transverse magnetic field. It is assumed that no external electric field is acting on the system and the magnetic Reynolds number is very small. The operational method is used to obtain exact solutions for the fluid and the particle velocities and the shear stress at the plate. Quantitative analysis of the results is made to disclose the simultaneous effects of the magnetic field and the particles on the fluid velocity and the wall shear stress.
Kannan and Venkataraman (2007) analyzed the viscous dusty fluid between two harmonically oscillating plates when a body force is applied at time $t = 0$ in the direction of the motion of the plates. It is inferred that growth of velocity of both the phases is rapid in the early stages but soon it tends to follow a steady pattern. When there is no body force, reverse type of flow occurs initially which shoots up rapidly to follow an oscillatory pattern.

Kannan and Venkataraman (2010) analytically studied the heat transfer rate and free convection in an infinite and porous medium. They discussed velocity and temperature fields by using Rayleigh number. They extend the results up to second order mean in both fields. It is observed that, because of relaxation time of dust, the second order mean flow occurred.
Venkataraman and Kannan (2010) has analyzed the two-phase fluid flow in the existence of Magnetic field. They evaluated the numerical values of exact solution. Also found the reversal particle flow occurred only when the Eckert number is eight. Also, in the presence of magnetic field, increase in mass concentration causes increase in fluid velocity and skin friction values decreases only when the rotation parameter values are decreased.

This present study investigates unsteady oscillatory flow of dusty viscoelastic electrically conductive fluid down an inclined parallel plate in presence of uniform magnetic field applied externally transverse to the direction of flow. Assuming the plates are maintained at temperatures which decay exponentially with time, the expressions for fluid velocity, dust particle velocity and temperature profile, fluid dust particle flux, heat transfer, viscous drag at the boundary layers are obtained.
The velocity profile of fluid and dust particle the temperature profile are shown graphically for different values of various physical parameters.

Basic Assumptions in Dusty Viscoelastic Fluid Flow

In order to formulate the fundamental equations of motion of the dusty viscoelastic fluid flows in a reasonably simple manner and to bring out the essential features, certain basic assumptions are made. They are as follows:

(i) The fluid is an incompressible non-Newtonian viscoelastic fluid.
(ii) The plates are infinitely long so that the fluid velocity (u_1) and dust particles velocity (u_1) are functions of $y \; and \; t$ only
(iii) There are neither chemical reactions, mass transfer nor heat radiation among the dust particles
(iv) The number density of dust particles is constant and has small value throughout the fluid motion
(v) Dust particles are solids, elastic spheres, identical and symmetrical in size, electrically non-conducting and are distributed uniformly within the fluid motion
(vi) Hall effect, polarization effect and the effect due to Buoyancy are negligible
(vii) Initially (i.e. at time $t = 0$) there is no flow and plates are at time different temperatures (i.e. at $t = 0, \; T = T_0 \; at \; y = -h \; and \; t > 0 \; T = T_1 \; at \; y = +h$)
(viii) The fluid under consideration is finitely conducting so that Joule heat due to the presence of external magnetic field is negligible
(ix) The value of magnetic Reynolds number (Re) is small enough so that the induced field is negligible

Basic Governing Equations

We consider fully developed flow of an incompressible oscillatory dusty Rivlin-Ericksen fluid of electrically conducting material through a parallel plate channel separated by $2h$, inclined horizontally by an angle θ. The upper and lower plate are kept stationary and maintained at two different but constant temperature with upper plate simultaneously subjected to a steady change is temperature. Let the central line of the channel as the $x-axis$ while $y-axis$ is perpendicular to it. The uniform magnetic field B_0 is applied normal to the plates. So that the velocity and magnetic field profile are $V = [u(y,t), 0, 0] \; and \; B = [0, B_0, 0]$. The inertial force experienced by the dust particles is equal and opposite to that experienced by the dust particles due to the fluid motion.
The governing equations under above assumptions are momentum equations for the fluid

$$\rho\left(\frac{\partial u_1}{\partial t} + \frac{\partial u_1}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u_1}{\partial y^2} + \alpha_1 \frac{\partial^3 u_1}{\partial t \partial y^2} + k_1 N(u_2 - u_1) - \sigma B_0^2 u_1 + g \sin\theta \qquad (1)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y}+g\cos\theta=0 \qquad (2)$$

Momentum equation for the dusty particle

$$m\frac{\partial u_2}{\partial t}-k(u_1-u_2)=0 \qquad (3)$$

Energy equation for fluid

$$\rho k_3\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=k_2\frac{\partial^2 T}{\partial y^2}=\mu\left(\frac{\partial u_1}{\partial y}\right)^2+\alpha_1\frac{\partial u_1}{\partial y}\frac{\partial^2 u_1}{\partial t\partial y} \qquad (4)$$

where p is the fluid pressure, M represent mass of dust particle, μ is the kinematic coefficient of fluid viscosity, α_1 means kinematic coefficient of viscoelasticity, N is the number density of the dust particles, k_1 is the proportionally constant, k_2 is the thermal conductivity of the fluid, k_3 is the specific heat at constant pressure. u_1 *and* u_2 represent the velocity of fluid and particles respectively and T is the temperature of the fluid. The fluid pressure P is define as

$$p=pg(x\sin\theta-y\cos\theta)+\rho xa(t)+A \qquad (5)$$

where A is a constant

Therefore, the boundary conditions can be deduced as

$$\begin{aligned} u_1&=u_2=0\,, \quad T=0 \quad at \quad y=-h \\ u_1&=u_2=0\,, \quad T=T_0e^{iwt} \quad at \quad y=h \end{aligned} \qquad (6)$$

Method of Solutions

The momentum and carrying equation the governing flow is simplified by writing the equation (1) – (5) in the non dimensional form. We define the following non-dimensional quantities as

$$u_1=\overline{u_1}u_0\,, \quad y=\overline{y}h\,, \quad x=\overline{x}h\,,\ u_2=\overline{u_2}v_0\,, \quad t=\frac{\overline{t}h}{u_0}\,, \quad \overline{T}=\frac{T}{T_0}$$

$$\lambda=\frac{u_0}{v_0}\,, \quad a=\frac{\overline{a}\,u_0^2}{h}$$

$$\frac{\partial u_1}{\partial t}+s\frac{\partial u_1}{\partial y}=-a(t)+\frac{1}{\mathrm{Re}}\frac{\partial^2 u_1}{\partial y^2}+\frac{\alpha\partial^3 u_1}{\partial t\partial y^2}+\frac{C_P}{R_P}\left(\frac{u_2}{\lambda}-u_1\right)-\frac{Ha}{\mathrm{Re}}u_1 \qquad (7)$$

$$R_P\frac{\partial u_2}{\partial t}-(\lambda u_1-u_2)=0 \qquad (8)$$

$$\frac{\partial T}{\partial t}+s\frac{\partial T}{\partial y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2}-\frac{Ec}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\alpha Ec}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial u_1}{\partial y}\frac{\partial^2 u_1}{\partial t\partial y}+\frac{HaEc}{\mathrm{Re}\,\mathrm{Pr}}u_1^2 \qquad (9)$$

The boundary conditions (6) becomes

$$\left.\begin{aligned} u_1&=u_2=0 \quad T=1 \quad at \quad y=1 \\ u_1&=u_2=0 \quad T=0 \quad at \quad y=-1 \end{aligned}\right\} \qquad (10)$$

where

$S=\dfrac{v}{u_0}$ Suction Parameter

$\mathrm{Re}=\dfrac{u_0 h\rho}{\mu}$ Reynolds Number

$$\alpha = \frac{\alpha_1}{\rho h^2} \qquad \text{Viscoelastic Parameter}$$

$$C_P = \frac{MN}{\rho} \qquad \text{Dust Particle Concentration}$$

$$R_P = \frac{Mu_0}{k_1 h} \qquad \text{Relaxation time Parameter of dust particle}$$

$$Ha = \sqrt{\frac{B_0^2 h^2 \sigma}{\rho\mu}} \qquad \text{Hartmann Number}$$

$$\Pr = \frac{\mu k_3}{k_2} \qquad \text{Prandtl Number}$$

$$Ec = \frac{\mu u_0^2}{\rho k_3 (T - T_0)} \qquad \text{Eckert Number}$$

For purely an oscillatory flow we take the pressure gradient of the

$$-a(t) = ce^{iwt} \tag{11}$$

where c is a constant and w is the frequency of oscillations.
Due to the selected of pressure gradient we assume the solution for equation (7) – (10) in the form.

$$u_1 = u_t e^{iwt} \tag{12}$$

$$u_2 = u_p e^{iwt} \tag{13}$$

$$T = T_f e^{iwt} \tag{14}$$

By substituting equation (12)-(14) into equation (7)-(10), we have

$$\frac{\partial^2 u_f}{\partial y^2} - A_1 \frac{\partial u_f}{\partial y} - A_2 u_f + A_3 = 0 \tag{15}$$

$$u_f(-1) = 0, \qquad u_f(1) = 0 \tag{16}$$

$$u_p = \frac{\lambda u_p}{A_4} \tag{17}$$

$$u_p(-1) = 0 \qquad u_p(1) = 0 \tag{18}$$

$$\frac{\partial^2 T_f}{\partial y^2} - A_5 \frac{\partial T_f}{\partial y} - A_6 T_f = 0 \tag{19}$$

$$T_f(-1) = 0 \qquad T_f(1) = 1 \tag{20}$$

The solution of equation (15) is give as

$$u_f(y) = c_1[\cosh(m_3 y) + \sinh(m_3 y)] + c_2[\cosh(m_4 y) + \sinh(m_4 y)] + \frac{A_3}{A_2} \tag{21}$$

where $m_3 = \frac{1}{2}\left(A_1 + \sqrt{A_1^2 + 4A_2}\right)$

$$m_4 = \frac{1}{2}\left(A_1 - \sqrt{A_1^2 + 4A_2}\right)$$

To obtain c_1 *and* c_2, substitute equation (16) into (21) to obtain the following

$$\begin{bmatrix} \cosh(m_3) - \sinh(m_3) & \cosh(m_4) - \sinh(m_4) \\ \cosh(m_3) + \sinh(m_3) & \cosh(m_4) + \sinh(m_4) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -\frac{A_3}{A_4} \\ -\frac{A_3}{A_4} \end{bmatrix} \tag{22}$$

Therefore

$$c_1 = \frac{-\sinh(m_4)\ A_3}{A_2\left[\cosh(m_3)\sinh(m_4) - \sinh(m_3)\cosh(m_4)\right]}$$

$$c_2 = \frac{A_3 \sinh(m_3)}{A_2\left[\cosh(m_3)\sinh(m_4) - \sinh(m_3)\cosh(m_4)\right]}$$

The solution of equation (17) is given as

$$u_p(y) = \lambda\left[c_1\left[\cosh(m_3 y) + \sinh(m_3 y)\right] + c_2\left[\cosh(m_4 y) + \sinh(m_4 y)\right] + \frac{A_3}{A_4}\right] \tag{23}$$

While solution of equation (19) is

$$T_f(y) = c_3\left[\cosh(m_5 y) + \sinh(m_5 y)\right] + c_4\left[\cosh(m_6 y) + \sinh(m_6 y)\right] \tag{24}$$

Where $m_5 = \frac{1}{2}\left(A_5 + \sqrt{A_5^2 + 4A_6}\right)$

$$m_6 = \frac{1}{2}\left(A_5 - \sqrt{A_5^2 + 4A_6}\right)$$

To obtain c_3 *and* c_4, substitute equation (20) into equation (24) to yield the following

$$\begin{bmatrix} \cosh(m_5) - \sinh(m_5) & \cosh(m_6) - \sinh(m_6) \\ \cosh(m_5) - \sinh(m_5) & \cosh(m_6) + \sinh(m_6) \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{25}$$

$$c_3 = \frac{1}{2}\,\frac{\cosh(m_6) - \sinh(m_6)}{\cosh(m_6)\sinh(m_5) - \sinh(m_6)\cosh(m_5)}$$

$$c_4 = -\frac{1}{2}\,\frac{\cosh(m_5) - \sinh(m_5)}{\cosh(m_6)\sinh(m_5) - \sinh(m_6)\cosh(m_5)}$$

Substituting equation (21) (23) and (24) into equation (12) – (14) to obtain the following

$$u_1 = \left[c_1\left[\cosh(m_3 y) + \sinh(m_3 y)\right] + c_2\left[\cosh(m_4 y) + \sinh(m_4 y)\right] + \frac{A_3}{A_2}\right]\cos(wt) \tag{26}$$

$$u_2 = \lambda\left[c_1\left[\cosh(m_3 y) + \sinh(m_3 y)\right] + c_2\left[\cosh(m_4 y) + \sinh(m_4 y)\right] + \frac{A_3}{A_2}\right]\cos(wt) \tag{27}$$

$$T = \left[c_3\left[\cosh(m_5 y) + \sinh(m_5 y)\right] + c_2\left[\cosh(m_6 y) + \sinh(m_6 y)\right]\right]\cos(wt) \tag{28}$$

Skin Friction

The viscous drag acting at the plates for fluid (τ_f) and for the particles (τ_p) are

$$\begin{aligned} \tau_f &= \left[\left(\frac{1}{R_e} - \alpha\frac{\partial}{\partial t}\right)\frac{\partial u_1}{\partial y}\right]_{y=\pm 1} \\ &= \left[\frac{1}{R_e}\frac{\partial u_1}{\partial y} - \alpha\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial y}\right)\right]_{y\pm 1} \end{aligned} \tag{29}$$

Substituting equation (26) into equation (29) to yield

$$\tau_f = \cos(wt) + R_e\alpha w\sin(wt)\left[\frac{c_2 m_4 e^{\pm m_4}}{R_e} + \frac{c_1 m_3 e^{\pm m_3}}{R_e}\right] \tag{30}$$

Also,

$$\tau_p = \left[\left(\frac{1}{R_e} - \alpha\frac{\partial}{\partial t}\right)\left(\frac{\partial u_2}{\partial y}\right)\right]_{y=\pm 1}$$
$$= \left[\frac{1}{R_e}\frac{\partial u_2}{\partial y} - \alpha\frac{\partial}{\partial t}\left(\frac{\partial u_2}{\partial y}\right)\right]_{y=\pm 1} \qquad (31)$$

Substituting equation (27) into equation (31) to yield

$$\tau_p = \cos(wt) + R_e w\alpha\sin(wt)\left[\frac{c_1\lambda m_3 e^{\pm m_3}}{R_e A_4} + \frac{c_2\lambda m_4 e^{\pm m_4}}{R_e A_4}\right] \qquad (32)$$

Flow Flux for Fluid and Particles

The flux of flow for fluid ϕ_f and the particles ϕ_p through the channel calculated as

$$\phi_f = \int_{-1}^{1} u_1 dy \qquad (33)$$

Substituting equation (26) into equation (33) to obtain

$$\phi_f = \frac{1}{R_2 m_3 m_4}\left[2\cos(wt)\left[c_1 A_2 m_4 \sinh(m_3) + c_2 A_2 m_3 \sinh(m_4) + A_3 m_3 m_4\right]\right] \qquad (34)$$

Also

$$\phi_p = \int_{-1}^{1} u_2 dy \qquad (35)$$

Substituting equation (27) into equation (35) to yield

$$\phi_p = \frac{1}{A_2 A_4 A_3 A_4}\left[2\lambda\cos(wt)\left[c_1 A_2 m_4 \sinh(m_3) + c_2 A_2 m_3 \sinh(m_4) + A_3 m_3 m_4\right]\right] \qquad (36)$$

Heat Transfer

The rate of heat transfer i.e. the heat transfer coefficient in terms of Nussett number (N_u) at the plate is given as

$$N_u = \left[\frac{dT}{dy}\right]_{y=\pm 1} \qquad (37)$$

Substituting equation (28) into equation (37) to obtain

$$N_u = \cos(wt)\left(c_3 m_5 e^{\pm m_5} + c_4 m_6 e^{\pm m_6}\right) \qquad (38)$$

Results and Discussion

This section discusses and presents the analytical results obtained through several graphs which demonstrate the effects of various parameters on the velocity profile of oscillatory Poiseullel flow of dusty viscoelastic fluid down an inclined channel.

Numerical evaluation for the analytical solution of this problem is performed and the results are illustrated graphically in figure 1-16 to show the interesting features of significant physical parameters on the velocity profile.

Figures 1-4 present the effect of magnetic parameter Ha on the velocity profile of fluid and dusty particles at $t = 0$ *and* $t = \frac{\pi}{3}$ respectively.

At $t = 0$ in figure1 and 2, the movement of fluid and particles in the channel slows down as the value of magnetic parameter increases. The same thing happened at $t = \frac{\pi}{3}$ as shown in figures 3 and 4. This is because magnetic field give rise to a resistive force, called the Lorentz force which acts opposite to the flow direction. From figure 9 and 10 it clearly revealed and the dust particle decreases as the viscoelastic parameter (α) increases at low magnetic field flux. But at high magnetic field flux, the velocity profile for both the fluid and particles further decreases and nearly reach its steady state as illustrated in figure 11 and 12.

Figures 5-8 show that the velocity profile for the fluid and the dust particles decreases monotonically with time at different values of time t from how magnetic field flux to higher magnetic field flux.

Figures 13-16 illustrate the effect of suction parameter (S) at low and high magnetic field flux on the movement of dusty fluid through the channel. It is observed from figures 13 and 14 that increasing the suction parameter decreases the velocity profile of the fluid and particles with the appearance of cross over of the velocity curve due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed.

Figure 15 and 16 shows that the suction has a more pronounced effect in the steady state of the velocity of both the fluid and particle at high magnetic field flux than low magnetic field flux.

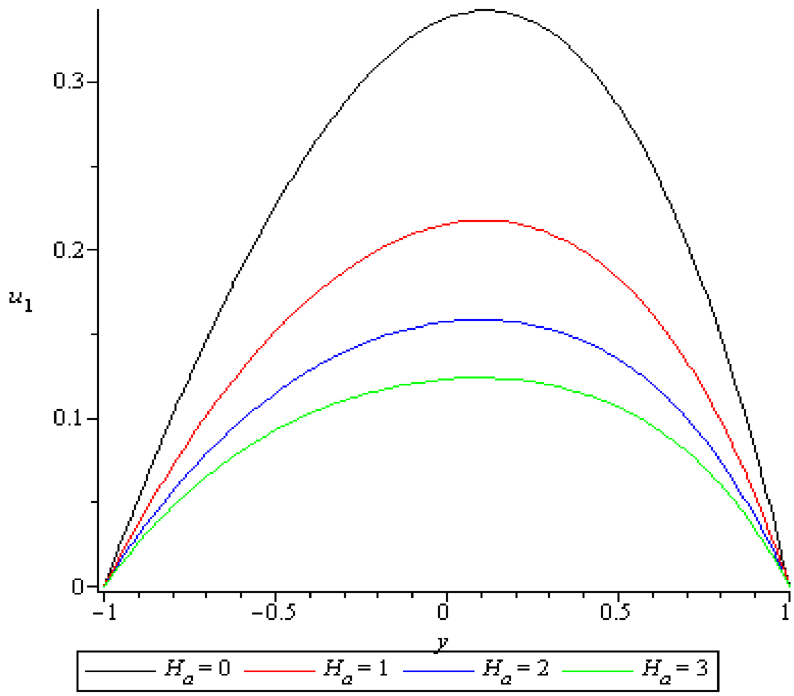

Figure 1: Fluid velocity profile ***for different values of*** Ha ***at*** $t=0$ ***where*** $S=1, R_e=0.5, \alpha=0.1, C_p=0.1, R_p=0.1, c=1, \lambda=1$ *and* $w=1$

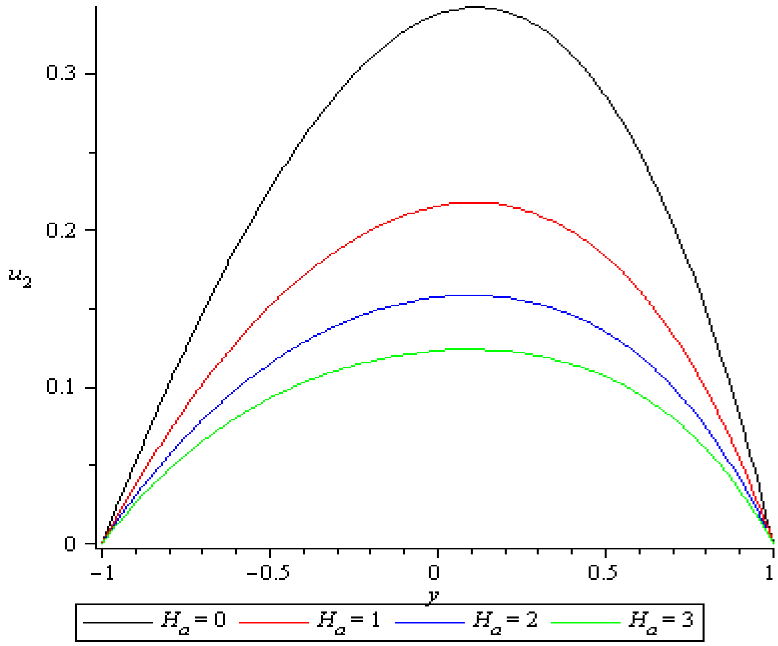

Figure 2: Particles velocity profile ***for different values of*** Ha ***at*** $t = 0$ ***where*** $S = 1,\ R_e = 0.5,\ \alpha = 0.1,\ C_p = 0.1,\ R_p = 0.1,\ c = 1,\ \lambda = 1$ *and* $w = 1$

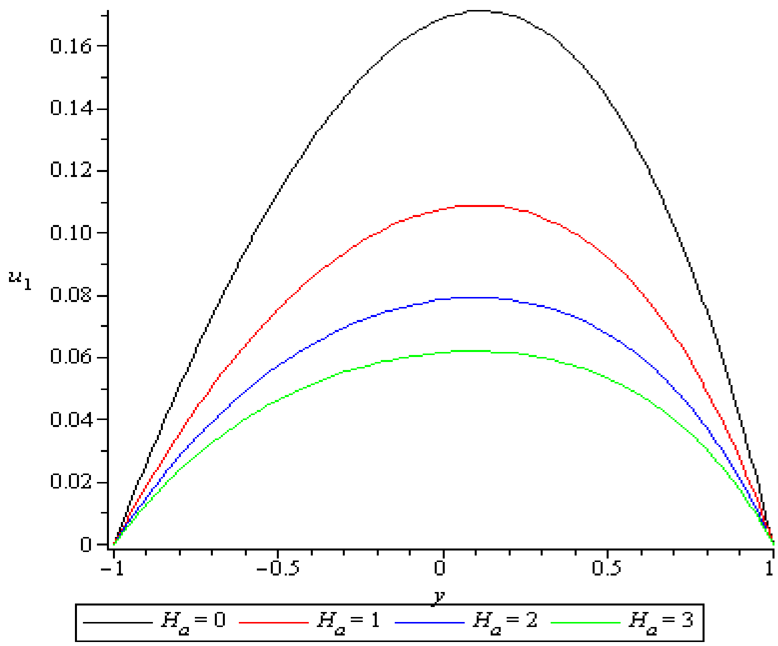

Figure 3: Fluid velocity profile ***for different values of*** Ha ***at*** $t = \pi/3$ ***then*** $S = 1,\ R_e = 0.5,\ \alpha = 0.1,\ C_p = 0.1,\ R_p = 0.1,\ c = 1,\ \lambda = 1$ *and* $w = 1$

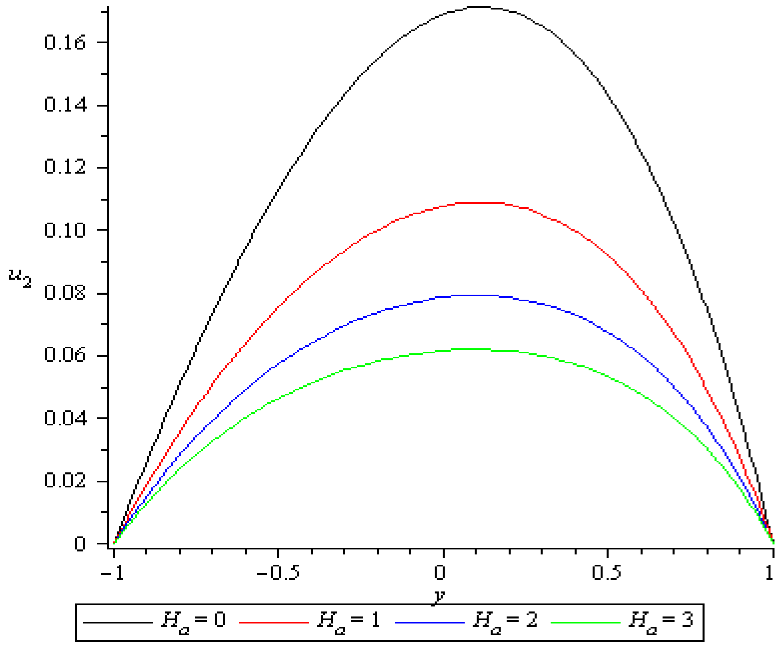

Figure 4: Particles velocity profile *for different values of* Ha ***at*** $t = \pi/3$ ***then*** $S = 1, R_e = 0.5, \alpha = 0.1, C_p = 0.1, R_p = 0.1, c = 1, \lambda = 1$ *and* $w = 1$

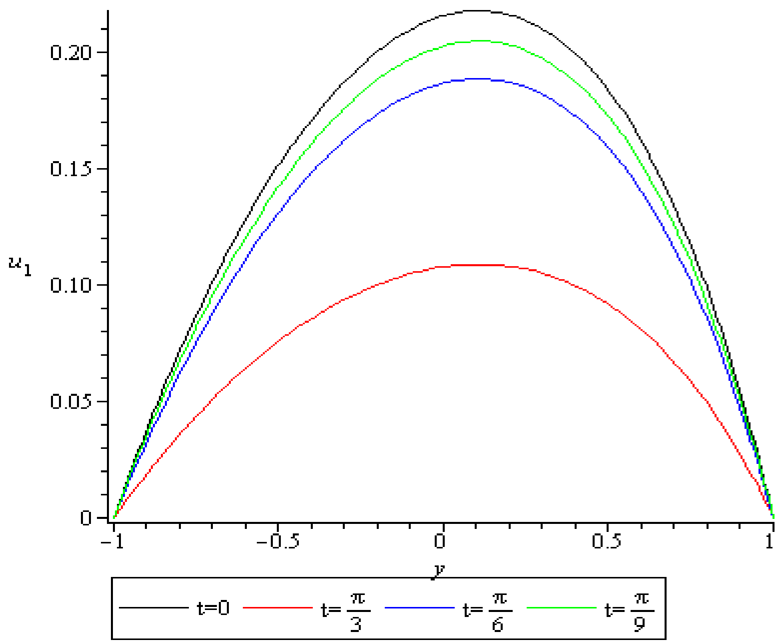

Figure 5: Fluid velocity profile *for different values of time* t ***at low magnetic field*** $Ha = 1$ ***when*** $S = 1, R_e = 0.5, \alpha = 0.1, C_p = 0.1, R_p = 0.1, c = 1, \lambda = 1$ *and* $w = 1$

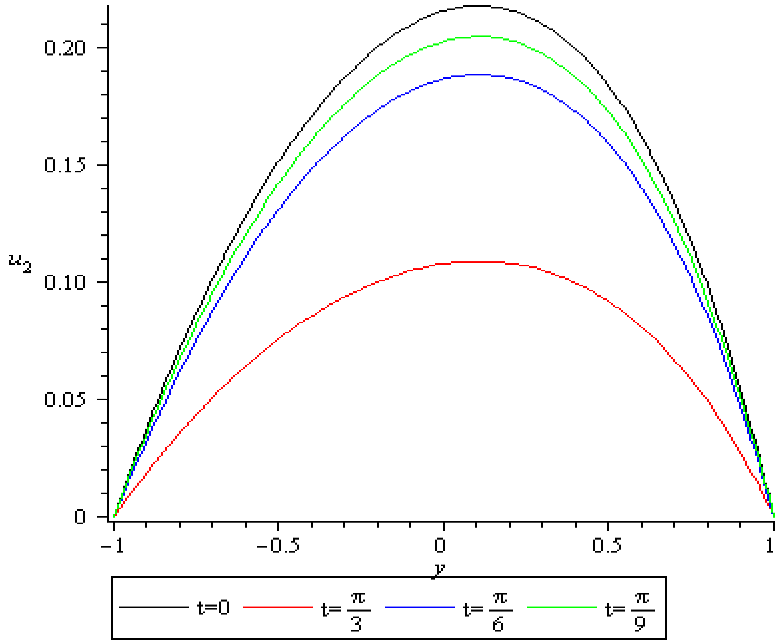

Figure 6: Particles velocity profile ***for different values of time*** t ***at low magnetic field*** $Ha = 1$ ***when*** $S = 1, R_e = 0.5, \alpha = 0.1, C_p = 0.1, R_p = 0.1, c = 1, \lambda = 1 \, and \, w = 1$

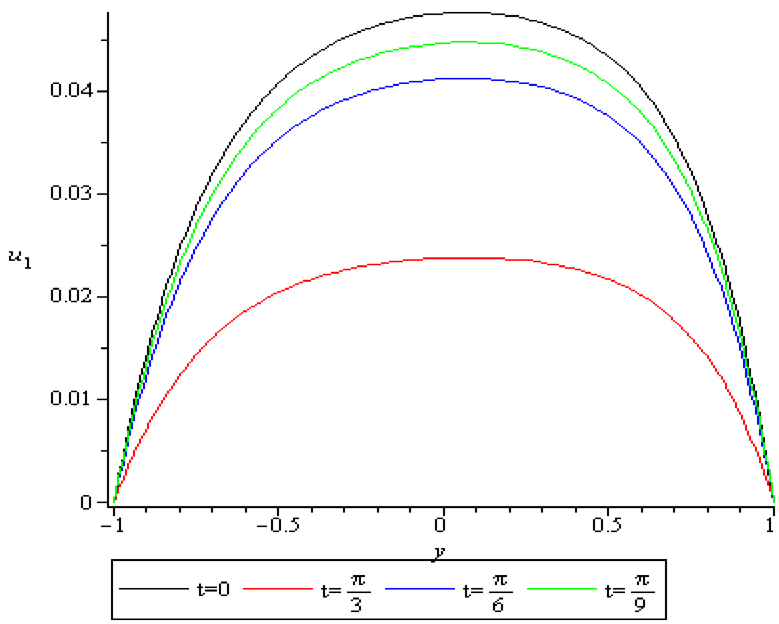

Figure 7: Fluid velocity profile ***for different values of time*** t ***at high magnetic field*** $Ha = 10$ ***when*** $S = 1, R_e = 0.5, \alpha = 0.1, C_p = 0.1, R_p = 0.1, c = 1, \lambda = 1 \, and \, w = 1$

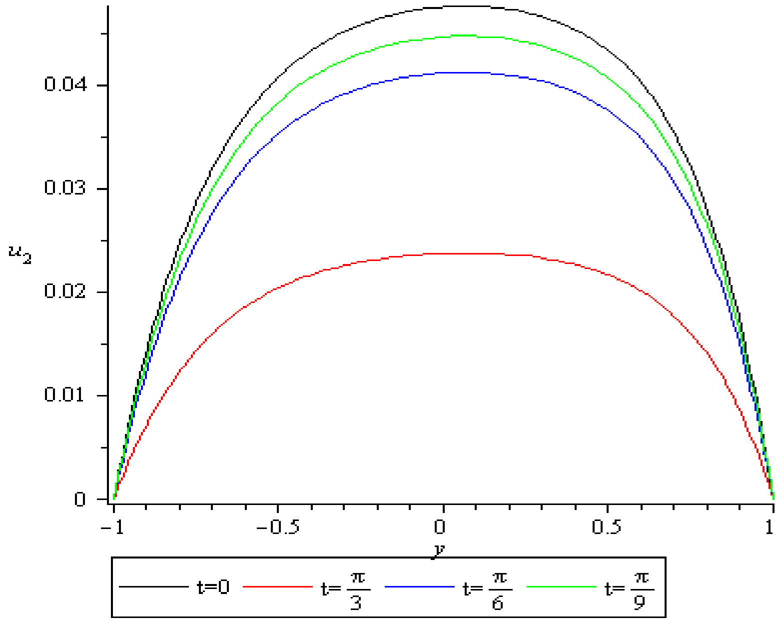

Figure 8: Particles velocity profile ***for different values of time*** t ***at high magnetic field*** $Ha = 10$ ***when*** $S = 1, R_e = 0.5, \alpha = 0.1, C_p = 0.1, R_p = 0.1, c = 1, \ \lambda = 1 \ and \ w = 1$

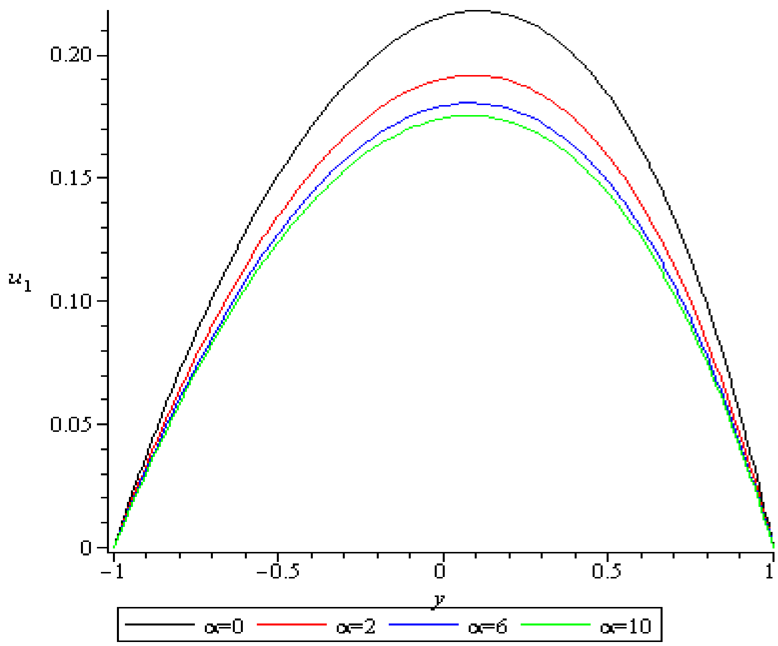

Figure 9: Fluid velocity profile ***for different values of*** α ***at low magnetic field*** $Ha = 1$ ***when*** $S = 1, R_e = 0.5, \alpha = 0.1, C_p = 0.1, R_p = 0.1, c = 1, \ \lambda = 1 \ and \ w = 1$

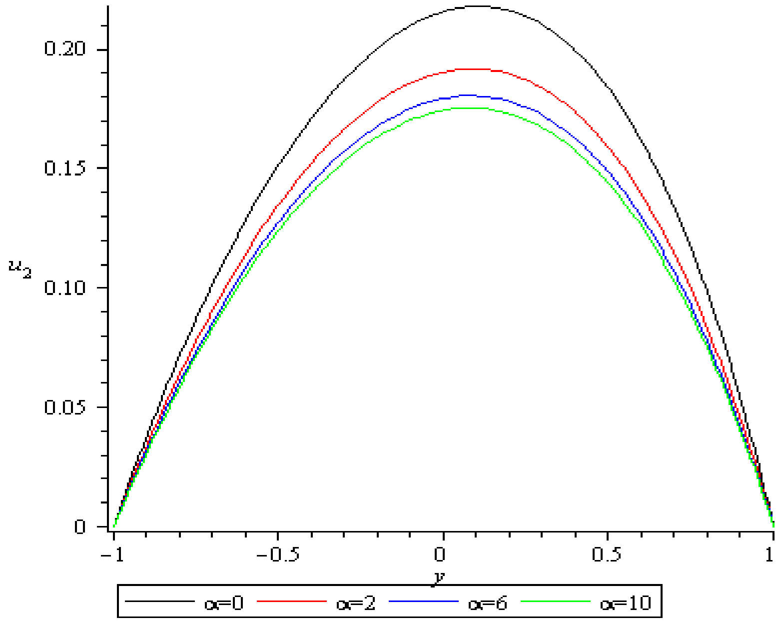

Figure 10: Particles velocity profile ***for different values of*** α ***at low magnetic field*** $Ha = 1$ ***when*** $S = 1, R_e = 0.5, \alpha = 0.1, C_p = 0.1, R_p = 0.1, c = 1, \lambda = 1$ *and* $w = 1$

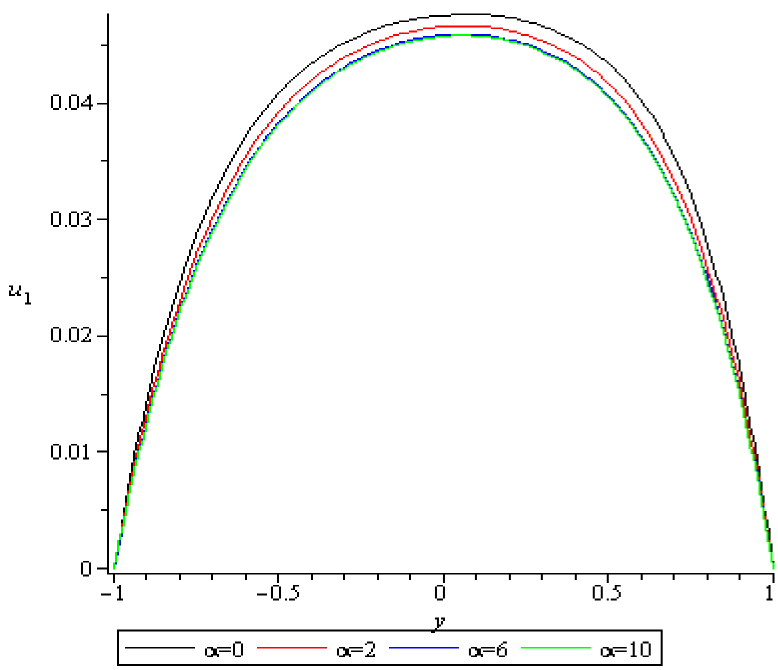

Figure 11: Fluid velocity profile ***for different values of*** α ***at high magnetic field*** $Ha = 10$ ***when*** $S = 1, R_e = 0.5, \alpha = 0.1, C_p = 0.1, R_p = 0.1, c = 1, \lambda = 1$ *and* $w = 1$

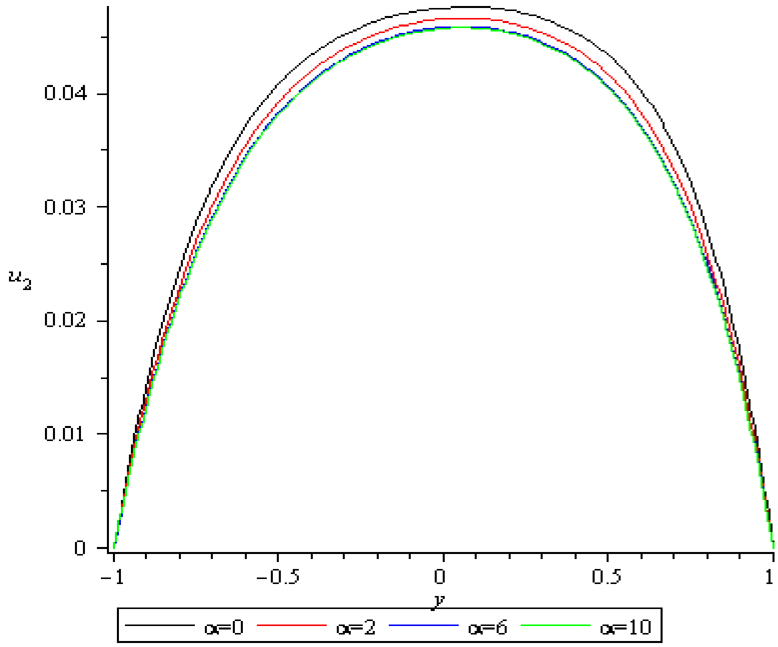

Figure 12: Particles velocity profile *for different values of* α *at high magnetic field* $Ha=10$ *when*
$S=1, R_e=0.5, \alpha=0.1, C_p=0.1, R_p=0.1, c=1,$ $\lambda=1$ *and* $w=1$

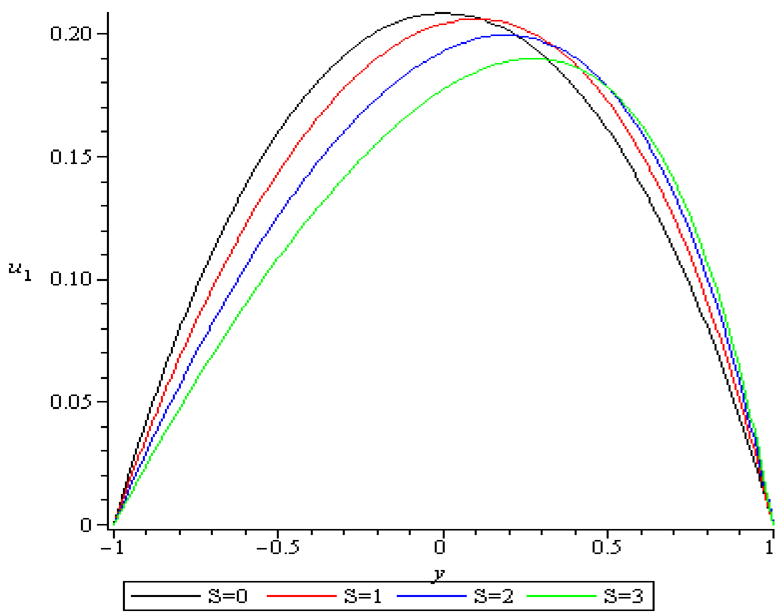

Figure 13: Fluid velocity profile *for different values of* S *at low magnetic field* $Ha=1$ *when*
$S=1, R_e=0.5, \alpha=0.1, C_p=0.1, R_p=0.1, c=1,$ $\lambda=1$ *and* $w=1$

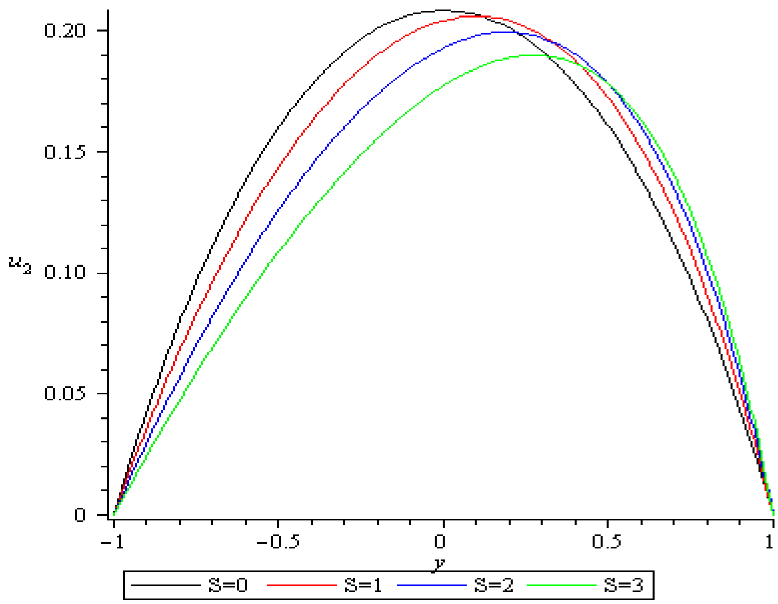

Figure 14: Particles velocity profile ***for different values of*** S ***at low magnetic field*** $Ha=1$ ***when*** $S=1, R_e=0.5, \alpha=0.1, C_p=0.1, R_p=0.1, c=1, \lambda=1$ *and* $w=1$

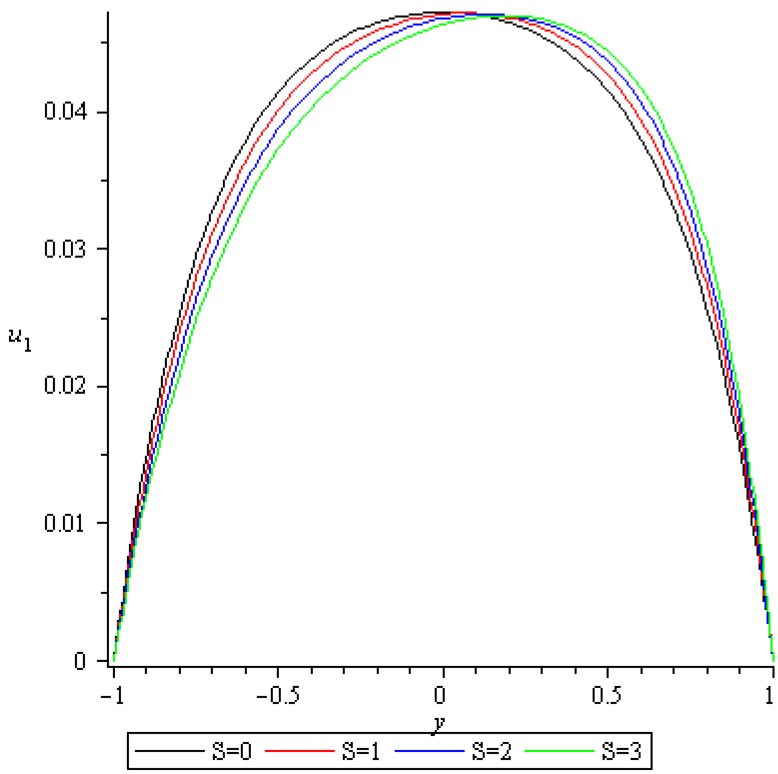

Figure 15: Fluid velocity profile ***for different values of*** S ***at high magnetic field*** $Ha=10$ ***when*** $S=1, R_e=0.5, \alpha=0.1, C_p=0.1, R_p=0.1, c=1, \lambda=1$ *and* $w=1$

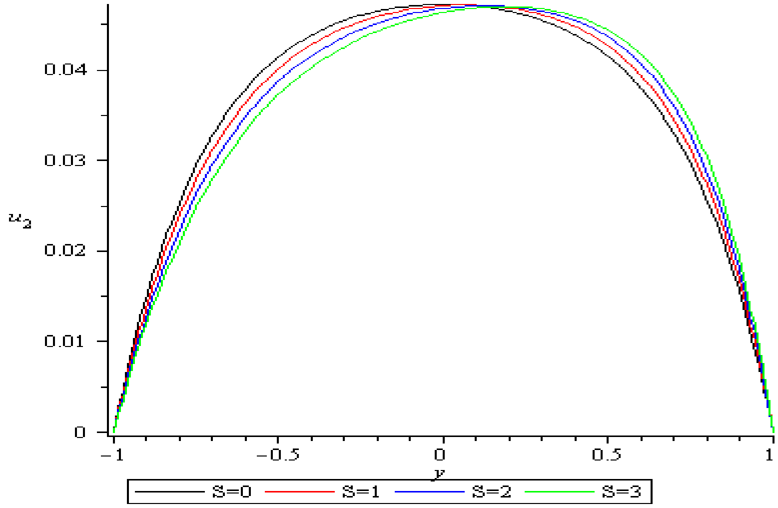

Figure 16: Particles velocity profile *for different values of* S *at high magnetic field* $Ha=10$ *when* $S=1, R_e=0.5, \alpha=0.1, C_p=0.1, R_p=0.1, c=1, \lambda=1$ *and* $w=1$

Conclusions

Analytical solutions are obtained for the velocity of the dusty viscoelastic fluid flow down inclined channel in the pressure of a uniform transverse magnetic field. The perturbation method is employed to solve the governing equation involved and the results are evaluated numerically and displayed graphically. In the light of the present investigation, following conclusions can be drawn:

(i) The dusty fluid velocity is almost the same with the dust particle velocity for all effects of this problem

(ii) Retarded motions of fluid and dust particles are seen during the growth of Hartmann number and suction parameter

(iii) The growth of viscoelasticity strengthens the shearing stresses at the two plates thereby reduced the movement of fluid and dust particles

(iv) Expression for viscous drag, the rate of heat transfer at the plates, flow flux for fluid and particle are obtained and presented.

Recommendations

The flow studied in this research is unsteady oscillatory Poisuelle flow of dusty viscoelastic fluid down an inclined channel. The fluid is electrically conducting and contain dusty particle. The equations governing the flow were solved analytically via perturbation techniques. Further study on this research can be on steady flow and method of solution governing the equation of the flow.

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